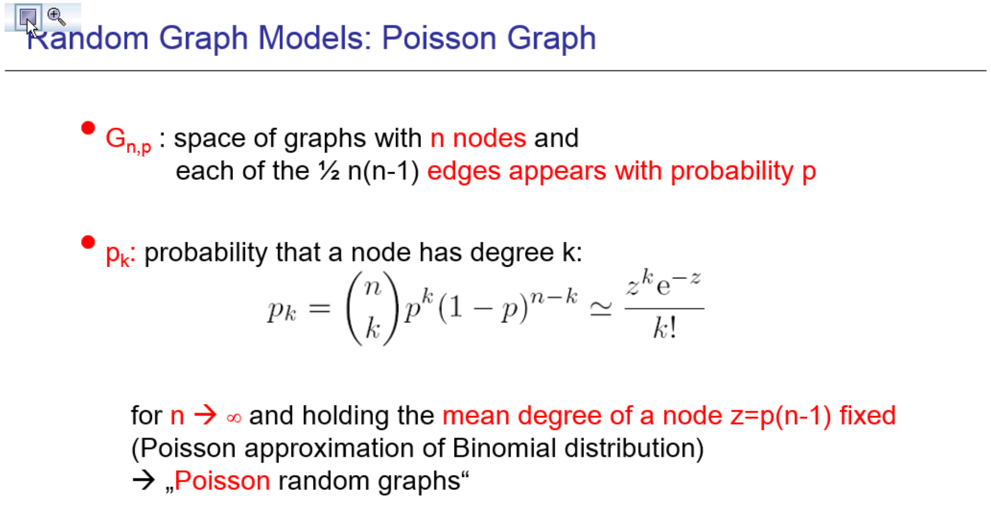
Script **generated by TTT**

Title: groh: profile1 (09.06.2015)

Date: Tue Jun 09 15:00:14 CEST 2015

Duration: 92:31 min

Pages: 98

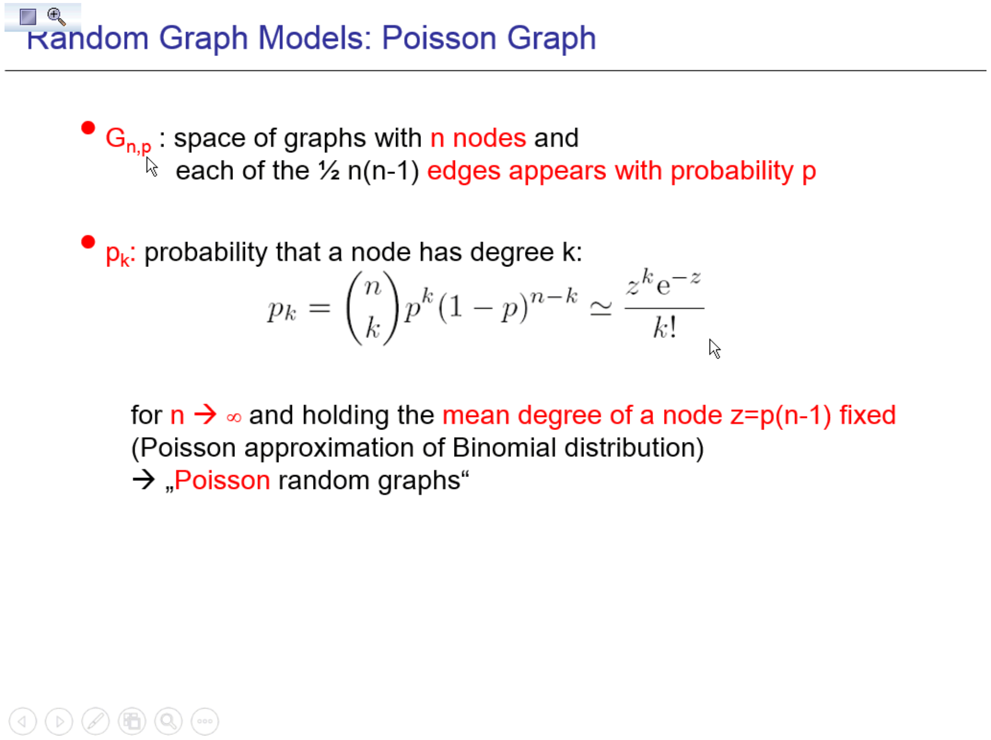


Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k \mathrm{e}^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

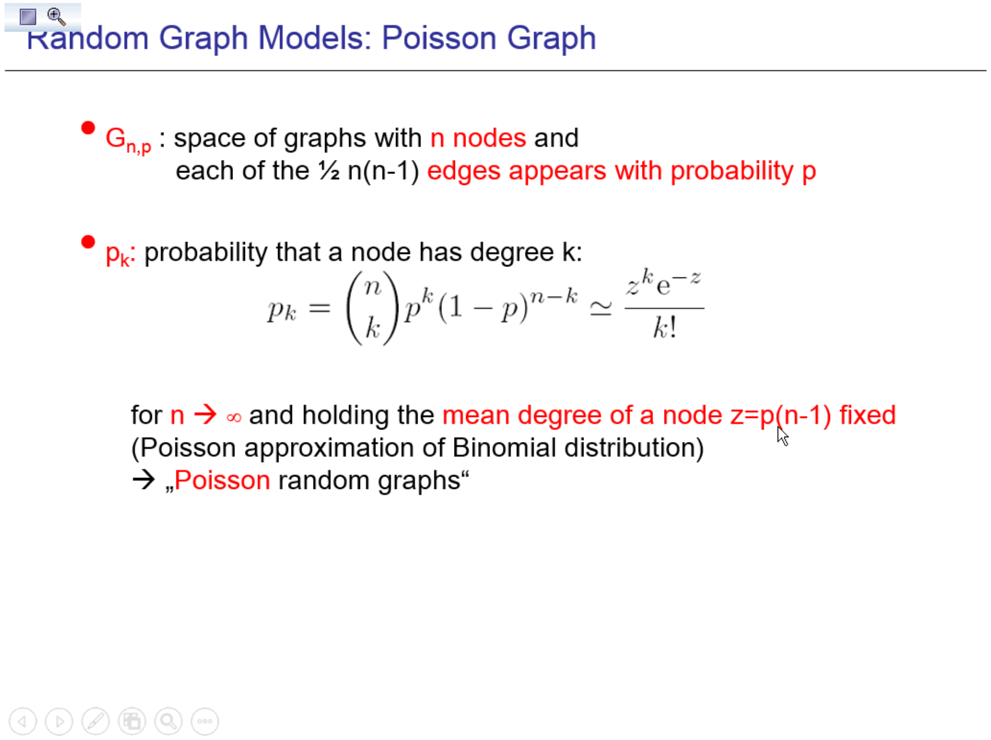


Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k \mathrm{e}^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“



Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k \mathrm{e}^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k \mathrm{e}^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k \mathrm{e}^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- Given: property Q („is connected“, „has diameter xyz“ etc.) of $G_{n,p}$:
  „$G_{n,p}$ has property Q with high probability“: $P(Q|n,p) \to 1$ iff $n \to \infty$
  (adaptated from [2] (which, in turn, is adaptated from [3]))
- In such models $G_{n,p}$ phase transitions exist for properties Q:
  „threshold function“ q(n) (with $q(n) \to \infty$ if $n \to \infty$) so that:

$$\lim_{n\to\infty} P(Q|n,p) = \begin{cases} 0 & \text{if } \lim_{n\to\infty} p(n)/q(n) = 0 \\ 1 & \text{if } \lim_{n\to\infty} p(n)/q(n) = \infty \end{cases}$$

(adaptated from [3])

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - \mathrm{e}^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - \mathrm{e}^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - \mathrm{e}^{-zS}$$

Random Graph Models: Poisson Graph

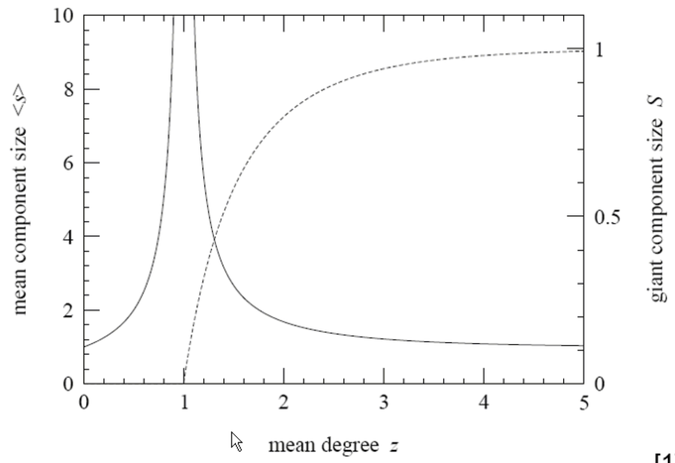
Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - \mathrm{e}^{-zS}$$

Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \dfrac{1}{1 - z + zS}$

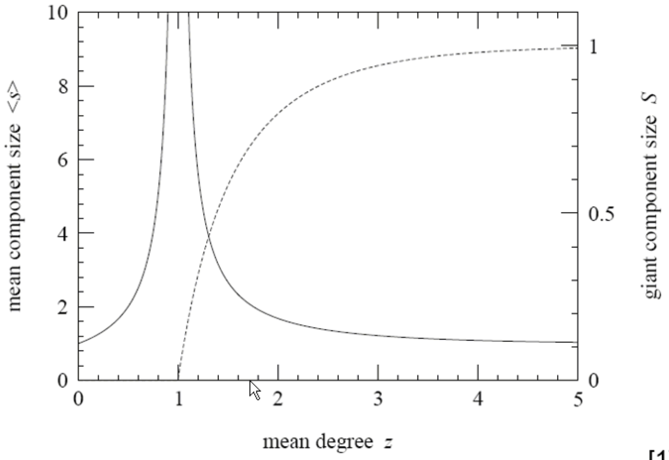


[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists

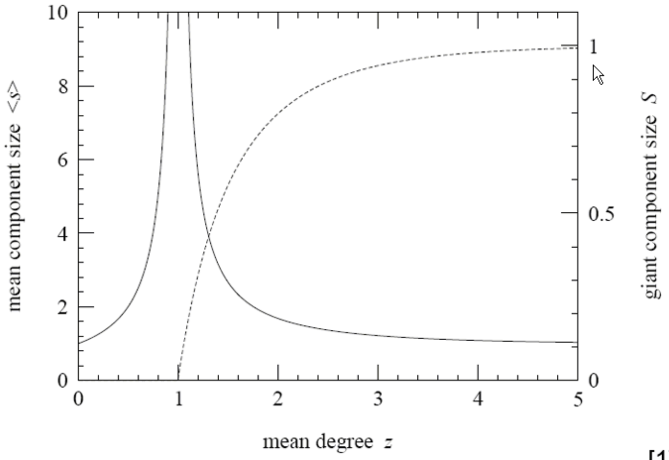
Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \dfrac{1}{1 - z + zS}$



[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n):   X exists

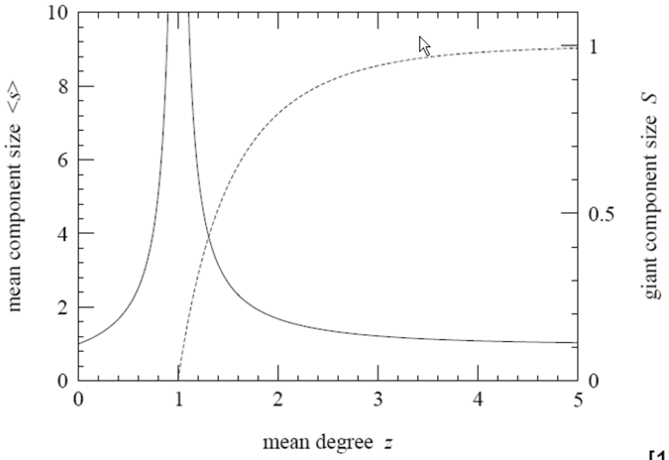


Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \dfrac{1}{1 - z + zS}$

[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n):   X exists

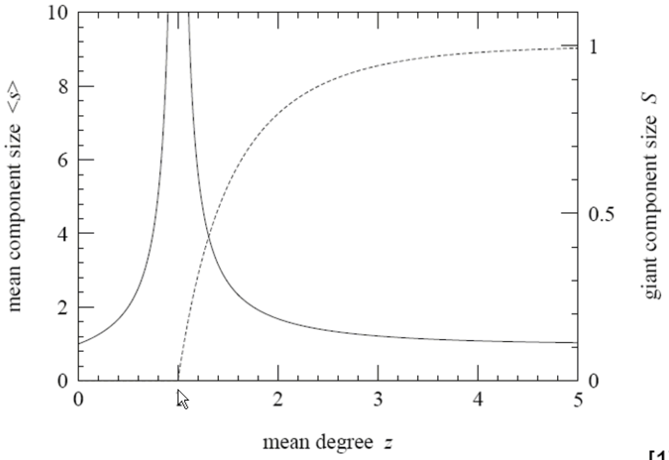


Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \dfrac{1}{1 - z + zS}$

[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n):   X exists



Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \dfrac{1}{1 - z + zS}$

[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n):   X exists

Random Graph Models: Poisson Graph

Very coarse (!!!) estimation of diameter l of $G_{n,p}$:

- average degree of nodes: z
  → in a distance of d from a node i should be approximately z^d many nodes
  → if $z^d = n$: $d = l$
  → $l \sim \log n / \log z \sim \log n$ (if z is kept constant)
- For a more exact derivation of the result see references in [1]
- We see: it is not difficult (in terms of how large must connectivity be) to achieve small diameters

Random Graph Models: Poisson Graph

Very coarse (!!!) estimation of diameter l of $G_{n,p}$:

- average degree of nodes: z
  → in a distance of d from a node i should be approximately z^d many nodes
  → if $z^d = n$: $d = l$
  → $l \sim \log n / \log z \sim \log n$ (if z is kept constant)
- For a more exact derivation of the result see references in [1]
- We see: it is not difficult (in terms of how large must connectivity be) to achieve small diameters

Random Graph Models: Poisson Graph

Unfortunately: small l is the only property in congruence with real world NW:

- Clustering coefficient $C^{(1)}$ of $G_{n,p}$:
  - Since $C^{(1)}$ is probability of transitivity and edges are "drawn" independently → $C^{(1)} = p = O(1/n)$ (if z is fixed, as usual)
  - C is usually much larger for real world NW:

	l (real)	l (random)	$C^{(2)}$ (real)	C (random)
Film collaboration	3.65	2.99	0.79	0.00027
Power Grid	18.7	12.4	0.08	0.005
C.elegans	2.65	2.25	0.28	0.05

[4]

- Degree distribution is Poisson and not power law

Random Graph Models: Poisson Graph

Unfortunately: small l is the only property in congruence with real world NW:

- Clustering coefficient $C^{(1)}$ of $G_{n,p}$:
  - Since $C^{(1)}$ is probability of transitivity and edges are "drawn" independently → $C^{(1)} = p = O(1/n)$ (if z is fixed, as usual)
  - C is usually much larger for real world NW:

	l (real)	l (random)	$C^{(2)}$ (real)	C (random)
Film collaboration	3.65	2.99	0.79	0.00027
Power Grid	18.7	12.4	0.08	0.005
C.elegans	2.65	2.25	0.28	0.05

[4]

- Degree distribution is Poisson and not power law

Random Graph Models: Poisson Graph

Unfortunately: small l is the only property in congruence with real world NW:

- Clustering coefficient $C^{(1)}$ of $G_{n,p}$:
  - Since $C^{(1)}$ is probability of transitivity and edges are "drawn" independently → $C^{(1)} = p = O(1/n)$ (if z is fixed, as usual)
  - C is usually much larger for real world NW:

	l (real)	l (random)	$C^{(2)}$ (real)	C (random)
Film collaboration	3.65	2.99	0.79	0.00027
Power Grid	18.7	12.4	0.08	0.005
C.elegans	2.65	2.25	0.28	0.05

[4]

- Degree distribution is Poisson and not power law

Random Graph Models: More Refined Models

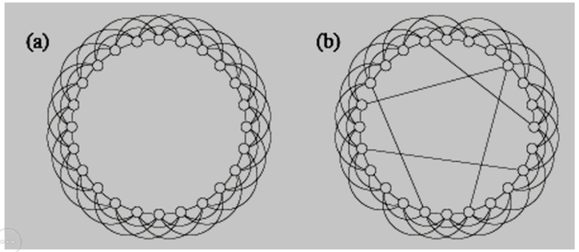
- Instead of having connection probability p as in Poisson $G_{n,p}$: demand certain degree distributions p_k (e.g. power law) → „configuration model"
- → results are promising but still not in full congruence with real world NW
- → still many difficult open problems
- still not accounted for: transitivity (high clustering coefficient)

Watts Strogatz Model

- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models
- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k
    → total number of edges = L k
  - „rewiring" of edges with probability p

before rewiring after rewiring

D = 1; k = 3



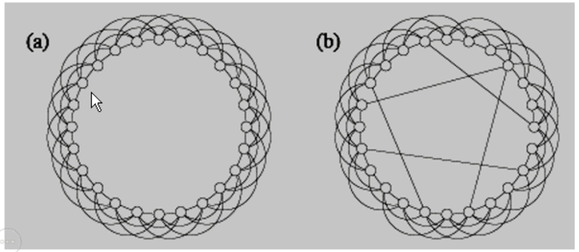
[1]

Watts Strogatz Model

- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models
- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k
    → total number of edges = L k
  - „rewiring" of edges with probability p

before rewiring after rewiring

D = 1; k = 3



[1]

Watts Strogatz Model

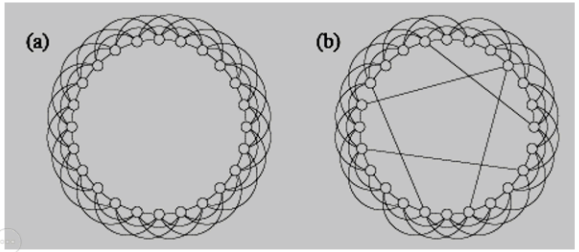
- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models

- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k

    → total number of edges = L k
  - „rewiring“ of edges with probability p

before rewiring after rewiring

D = 1; k = 3



[1]

Watts Strogatz Model

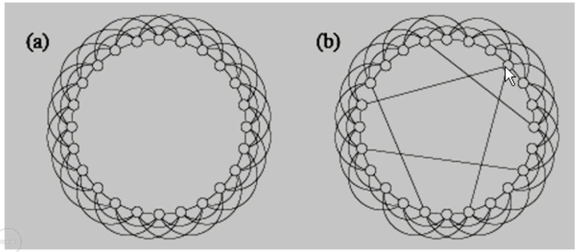
- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models

- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k

    → total number of edges = L k
  - „rewiring“ of edges with probability p

before rewiring after rewiring

D = 1; k = 3



[1]

Watts Strogatz Model

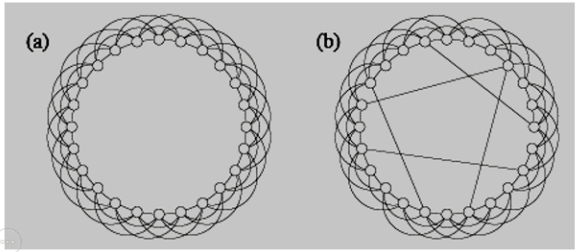
- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models

- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k

    → total number of edges = L k
  - „rewiring“ of edges with probability p

before rewiring after rewiring

D = 1; k = 3



[1]

Watts Strogatz Model

p: transition between regular lattice and sth. like a random graph: (for D=1:)

- p=0: regular lattice:
  - $C = C^{(1)} = (3k-3)/(4k-2)$ → ¾ for $k \to \infty$ → clustering coeff. „ok“
  - $l = L / 4k$ for $L \to \infty$ → no small world effect

- p=1: similar to a random graph:
  - $C \sim 2k / L$ for $L \to \infty$ → clustering coeff too small
  - $l = \log L / \log k$ for $L \to \infty$ → small world effect.

Watts Strogatz Model

p: transition between regular lattice and sth. like a random graph:
(for D=1:)

- p=0: regular lattice:
  - $C = C^{(1)} = (3k-3)/(4k-2)$ → ¾ for $k \to \infty$ → clustering coeff. „ok“
  - $l = L / 4k$ for $L \to \infty$ → no small world effect
- p=1: similar to a random graph:
  - $C \sim 2k / L$ for $L \to \infty$ → clustering coeff too small
  - $l = \log L / \log k$ for $L \to \infty$ → small world effect.

Watts Strogatz Model

p: transition between regular lattice and sth. like a random graph:
(for D=1:)

- p=0: regular lattice:
  - $C = C^{(1)} = (3k-3)/(4k-2)$ → ¾ for $k \to \infty$ → clustering coeff. „ok“
  - $l = L / 4k$ for $L \to \infty$ → no small world effect
- p=1: similar to a random graph:
  - $C \sim 2k / L$ for $L \to \infty$ → clustering coeff too small
  - $l = \log L / \log k$ for $L \to \infty$ → small world effect.

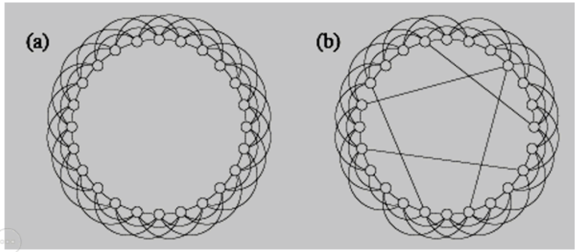
Watts Strogatz Model

- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models
- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k

    → total number of edges = L k
  - „rewiring“ of edges with probability p

before rewiring after rewiring

D = 1; k = 3



[1]

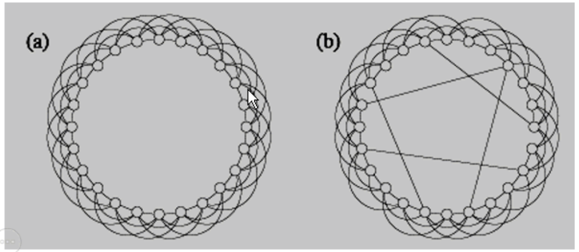
Watts Strogatz Model

- Great problem of random graphs: high clustering coeff. / transitivity does not occur for simple models
- → Watts & Strogatz 1998: Small World Model
  - L nodes in regular D-dim. lattice + periodic boundary cond.; D=1: Ring
  - each node connected to neighbors in lattice at distance of most k

    → total number of edges = L k
  - „rewiring“ of edges with probability p

before rewiring after rewiring

D = 1; k = 3



[1]

Watts Strogatz Model

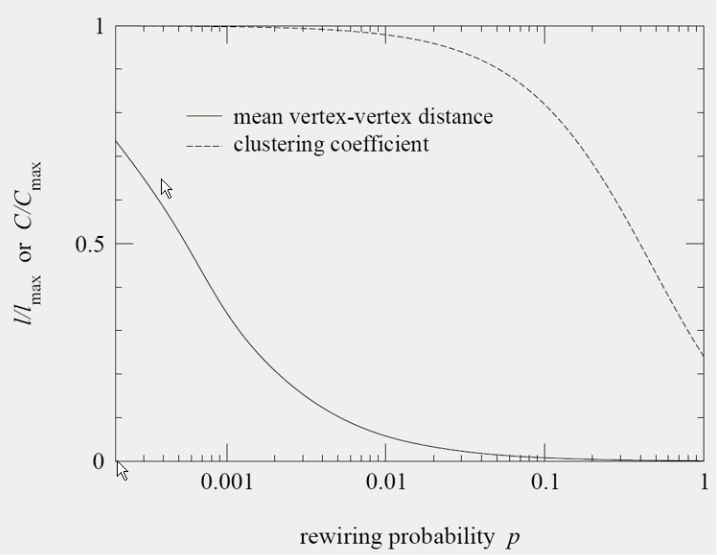
p: transition between regular lattice and sth. like a random graph:
(for D=1:)

- p=0: regular lattice:
  - $C = C^{(1)} = (3k-3)/(4k-2)$ → ¾ for k→∞ → clustering coeff. „ok“
  - $l = L / 4k$ for L→∞ → no small world effect

- p=1: similar to a random graph:
  - $C \sim 2k / L$ for L→∞ → clustering coeff too small
  - $l = \log L / \log k$ for L→∞ → small world effect.

Watts Strogatz Model

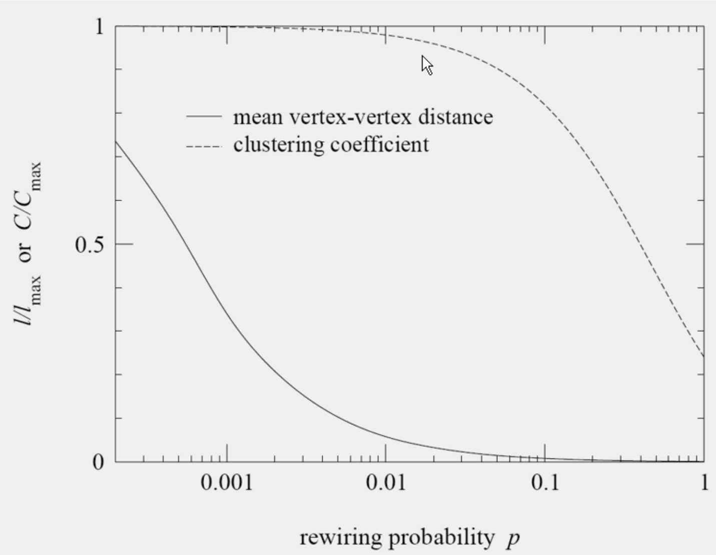
- Interesting area: intermediate values for p:



[1]

Watts Strogatz Model

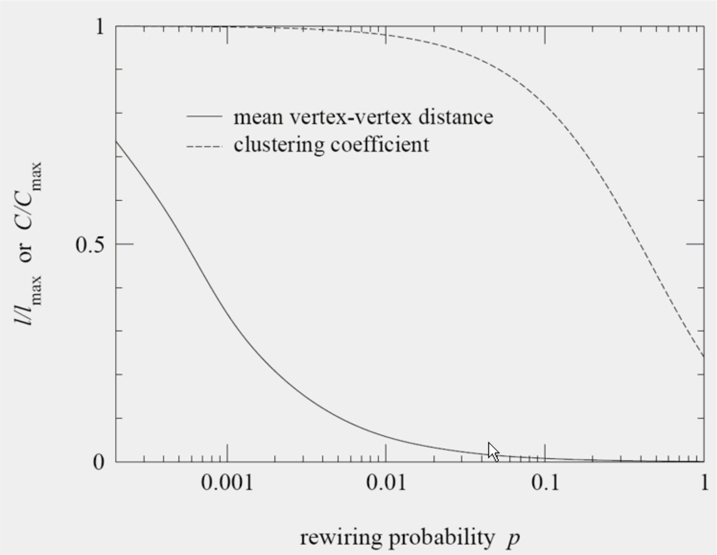
- Interesting area: intermediate values for p:



[1]

Watts Strogatz Model

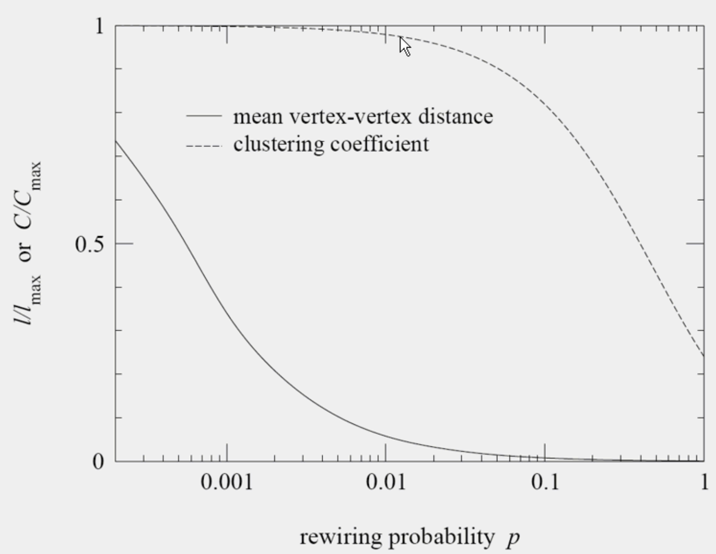
- Interesting area: intermediate values for p:



[1]

Watts Strogatz Model

- Interesting area: intermediate values for p:



[1]

Watts Strogatz Model

- Variants: -(1)- rewire both „ends“ of edges + allow self-edges +....
  → math.easier

  -(2)- only add **additional** shortcut edges (no rewiring)

- For (2):
  - mean total number of shortcuts = L k p
  - mean degree of each node = 2k(1+p)

Watts Strogatz Model

- Variants: -(1)- rewire both „ends“ of edges + allow self-edges +....
  → math.easier

  -(2)- only add **additional** shortcut edges (no rewiring)

- For (2):
  - mean total number of shortcuts = L k p
  - mean degree of each node = 2k(1+p)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  -- mean number of added shortcuts == Lkp
  -- and the mean degree == 2k + 2kp
  (2k from lattice plus 2kp added random shortcuts)
  -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$
  → $\tilde{p} = \dfrac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  - -- mean number of added shortcuts == Lkp
  - -- and the mean degree == 2k + 2kp
    (2k from lattice plus 2kp added random shortcuts)
  - -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$

  → $\tilde{p} = \frac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  - -- mean number of added shortcuts == Lkp
  - -- and the mean degree == 2k + 2kp
    (2k from lattice plus 2kp added random shortcuts)
  - -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$

  → $\tilde{p} = \frac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  - -- mean number of added shortcuts == Lkp
  - -- and the mean degree == 2k + 2kp
    (2k from lattice plus 2kp added random shortcuts)
  - -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$

  → $\tilde{p} = \frac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  - -- mean number of added shortcuts == Lkp
  - -- and the mean degree == 2k + 2kp
    (2k from lattice plus 2kp added random shortcuts)
  - -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$

  → $\tilde{p} = \frac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  -- mean number of added shortcuts == Lkp
  -- and the mean degree == 2k + 2kp
  (2k from lattice plus 2kp added random shortcuts)
  -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$
  → $\tilde{p} = \dfrac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  -- mean number of added shortcuts == Lkp
  -- and the mean degree == 2k + 2kp
  (2k from lattice plus 2kp added random shortcuts)
  -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$
  → $\tilde{p} = \dfrac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  -- mean number of added shortcuts == Lkp
  -- and the mean degree == 2k + 2kp
  (2k from lattice plus 2kp added random shortcuts)
  -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$
  → $\tilde{p} = \dfrac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  -- mean number of added shortcuts == Lkp
  -- and the mean degree == 2k + 2kp
  (2k from lattice plus 2kp added random shortcuts)
  -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$
  → $\tilde{p} = \dfrac{2kp}{L}$

(*): original model; (**) variant (2)

Watts Strogatz Model

- Degree distribution for variant (2):

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

for $j \geq 2k$, and $p_j = 0$ for $j < 2k$.

- in variant (2): p defined so that :
  -- mean number of added shortcuts == Lkp
  -- and the mean degree == 2k + 2kp
  (2k from lattice plus 2kp added random shortcuts)
  -- number of shortcuts is binomially distrib.

- Expectation of Binomial distribution: $E(X \sim B(L,\tilde{p})) = L\,\tilde{p}$
  → $\tilde{p} = \dfrac{2kp}{L}$

Watts Strogatz Model

- Degree distribution for original model (without proof):

$$p_j = \sum_{n=0}^{\min(j-k,k)} \binom{k}{n}(1-p)^n p^{k-n}\frac{(pk)^{j-k-n}}{(j-k-n)!}\,\mathrm{e}^{-pk} \qquad (*)$$

for $j \geq k$, and $p_j = 0$ for $j < k$.

Watts Strogatz Model

- Degree distribution for original model (without proof):

$$p_j = \sum_{n=0}^{\min(j-k,k)} \binom{k}{n}(1-p)^n p^{k-n}\frac{(pk)^{j-k-n}}{(j-k-n)!}\,\mathrm{e}^{-pk} \qquad (*)$$

for $j \geq k$, and $p_j = 0$ for $j < k$.

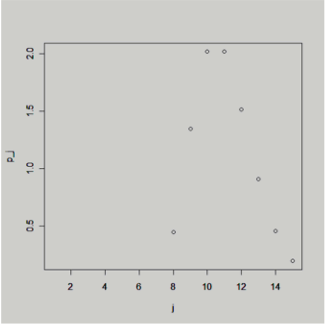
Watts Strogatz Model

- Degree distribution:

$$p_j = \binom{L}{j-2k}\left[\frac{2kp}{L}\right]^{j-2k}\left[1-\frac{2kp}{L}\right]^{L-j+2k} \qquad (**)$$

- Poisson approximation (justified):

$$p_j = \exp(-2kp)\frac{(2kp)^{j-2k}}{(j-2k)!}$$



k=3,p=0.5

- → almost constant
  → not in congruence with real world NW (power laws etc.)

Watts Strogatz Model

- Approximation (2) :

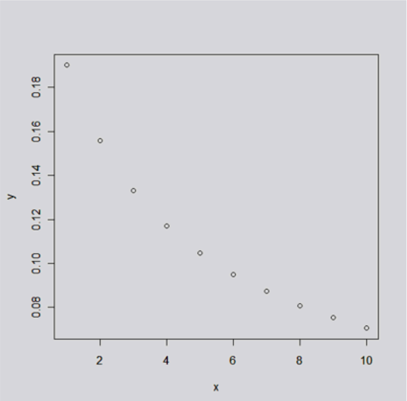
$$\ell = \frac{L}{k} f(\underbrace{Lkp}_{\text{mean number of shortcuts}})$$

== up to factor k same as Approx (1) for

$$\xi = 1/kp \text{ and } g(x) = x f(x)$$

independent investigations yield approximation

$$f(x) = \frac{1}{2\sqrt{x^2+2x}} \tanh^{-1} \sqrt{\frac{x}{x+2}}$$



Watts Strogatz Model

- Results for D > 1 : qualitatively similar

Models of Network Growth

- Random Graphs, Watts-Strogatz etc: Models aimed at reproducing properties of real world NW;

BUT: not really generative models / models of network growth

- → Models of Price and Barabasi & Albert

Watts Strogatz Model

- Results for D > 1 : qualitatively similar

Models of Network Growth

- Random Graphs, Watts-Strogatz etc: Models aimed at reproducing properties of real world NW;

BUT: not really generative models / models of network growth

- → Models of Price and Barabasi & Albert

Price‘s Model

- Basic principle:

„the rich get richer“

„Matthew effect“ („For to every one that hath shall be given...“ Bible: Mt25:29)

„preferential attachment“

- Assume directed citation NW:
  - p_k: fraction of nodes with in-degree k,
  - each node (paper) has av. out degree m
  - mean out-deg. $\overset{!}{=}$ mean in-deg. → $\sum_k kp_k = m$

- iteratively build graph by adding new vertices (and associated directed (out)edges from these nodes)

Price‘s Model

- Basic principle:
  - „the rich get richer“
  - „Matthew effect“ („For to every one that hath shall be given...“ Bible: Mt25:29)
  - „preferential attachment“
- Assume directed citation NW:
  - p_k: fraction of nodes with in-degree k,
  - each node (paper) has av. out degree m
  - mean out-deg. $\overset{!}{=}$ mean in-deg. → $\sum_k kp_k = m$
- iteratively build graph by adding new vertices (and associated directed (out)edges from these nodes)

Price‘s Model

- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ k_0=1
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

  - Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1} m$$

  - mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

Price‘s Model

- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ k_0=1
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

  - Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1} m$$

  - mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

Price‘s Model

- Basic principle:
  - „the rich get richer“
  - „Matthew effect“ („For to every one that hath shall be given...“ Bible: Mt25:29)
  - „preferential attachment“
- Assume directed citation NW:
  - p_k: fraction of nodes with in-degree k,
  - each node (paper) has av. out degree m
  - mean out-deg. $\overset{!}{=}$ mean in-deg. → $\sum_k kp_k = m$
- iteratively build graph by adding new vertices (and associated directed (out)edges from these nodes)

Price‘s Model

- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ k_0=1
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

  - Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1} m$$

  - mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

Price‘s Model

- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ k_0=1
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

  - Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1} m$$

  - mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

Price‘s Model

- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ k_0=1
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

  - Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1} m$$

  - mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

Price‘s Model

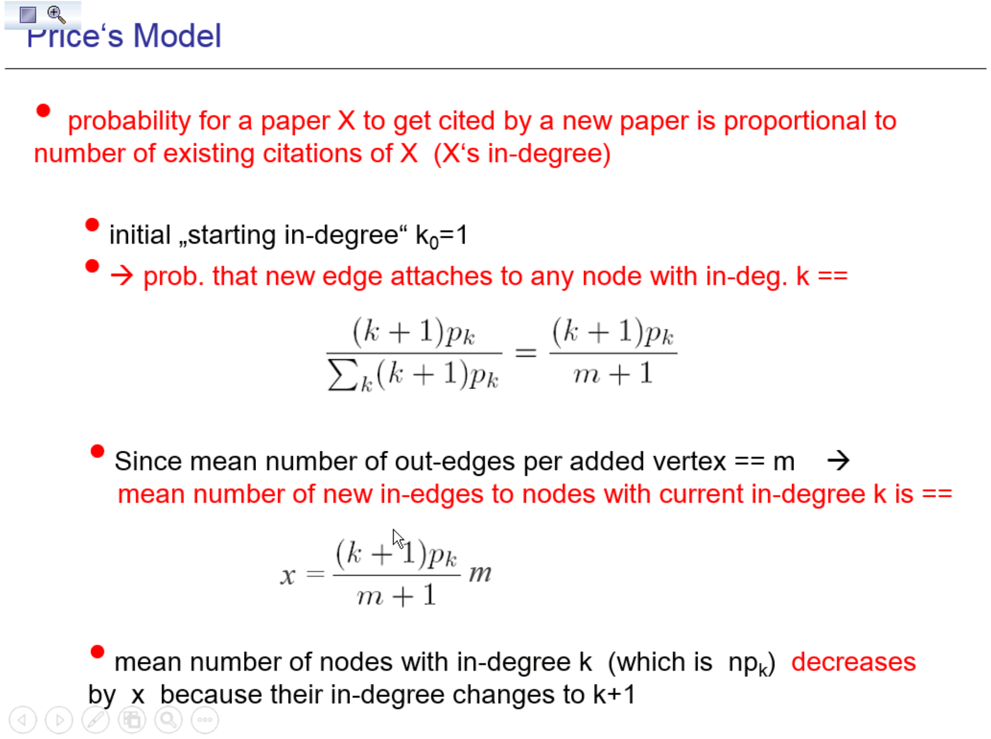
- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ k_0=1
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

  - Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1} m$$

  - mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1



Price‘s Model

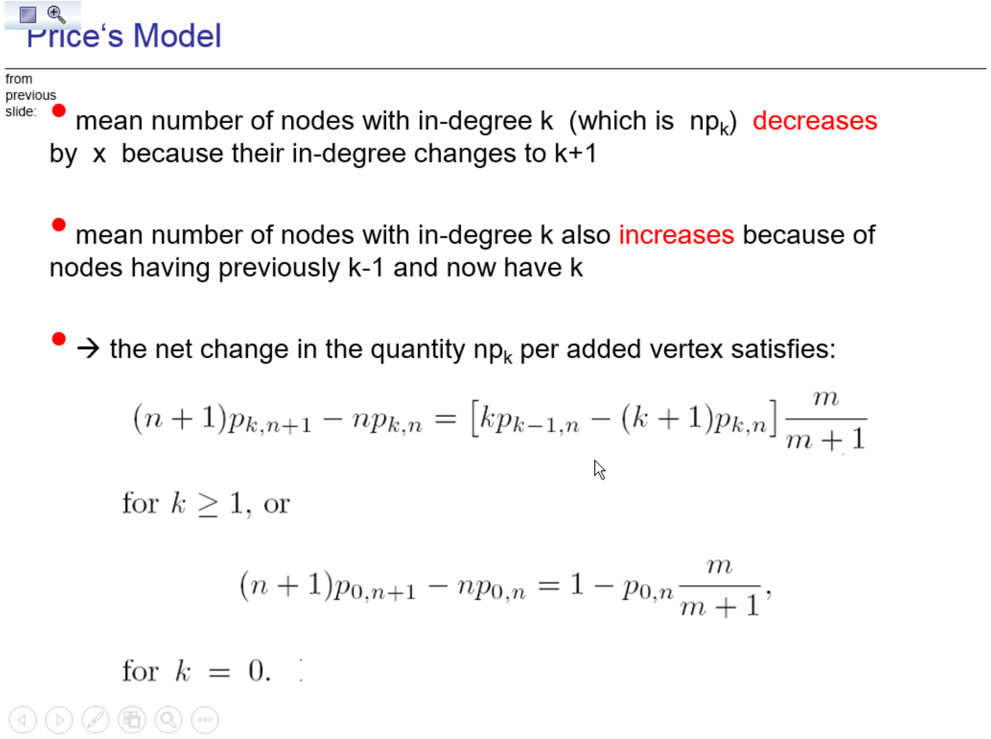
- probability for a paper X to get cited by a new paper is proportional to number of existing citations of X (X‘s in-degree)
  - initial „starting in-degree“ $k_0=1$
  - → prob. that new edge attaches to any node with in-deg. k ==

$$\frac{(k+1)p_k}{\sum_k (k+1)p_k} = \frac{(k+1)p_k}{m+1}$$

- Since mean number of out-edges per added vertex == m → mean number of new in-edges to nodes with current in-degree k is ==

$$x = \frac{(k+1)p_k}{m+1}\, m$$

- mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1



Price‘s Model

from previous slide:

- mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

- mean number of nodes with in-degree k also increases because of nodes having previously k-1 and now have k

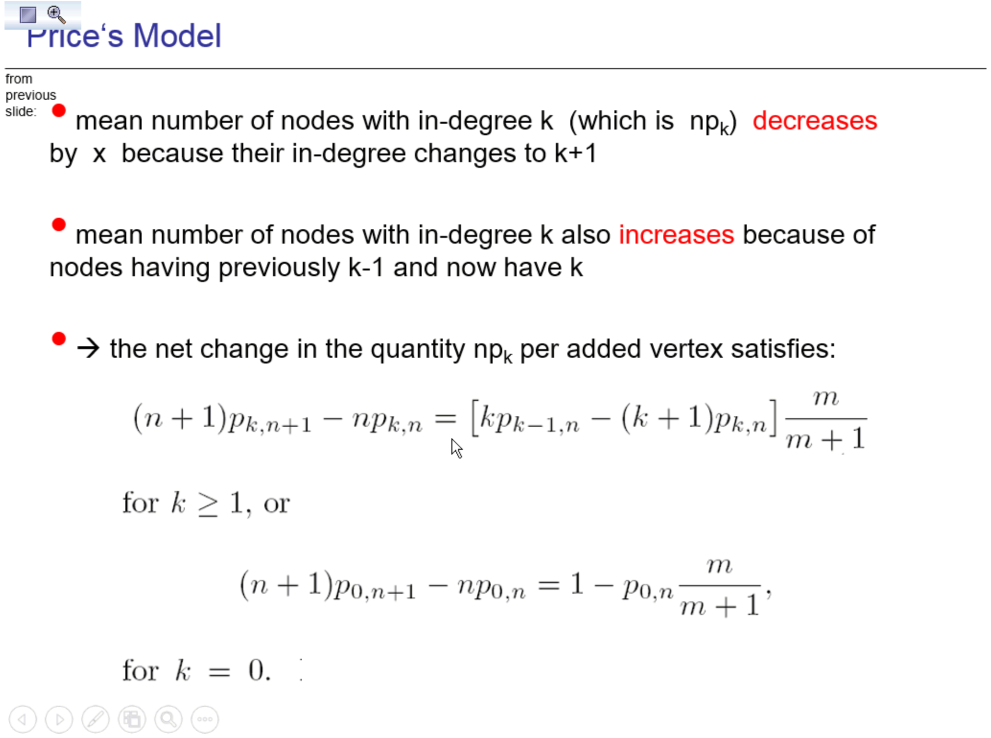
- → the net change in the quantity np_k per added vertex satisfies:

$$(n+1)p_{k,n+1} - np_{k,n} = \left[kp_{k-1,n} - (k+1)p_{k,n}\right]\frac{m}{m+1}$$

for $k \geq 1$, or

$$(n+1)p_{0,n+1} - np_{0,n} = 1 - p_{0,n}\frac{m}{m+1},$$

for $k = 0$.



Price‘s Model

from previous slide:

- mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

- mean number of nodes with in-degree k also increases because of nodes having previously k-1 and now have k

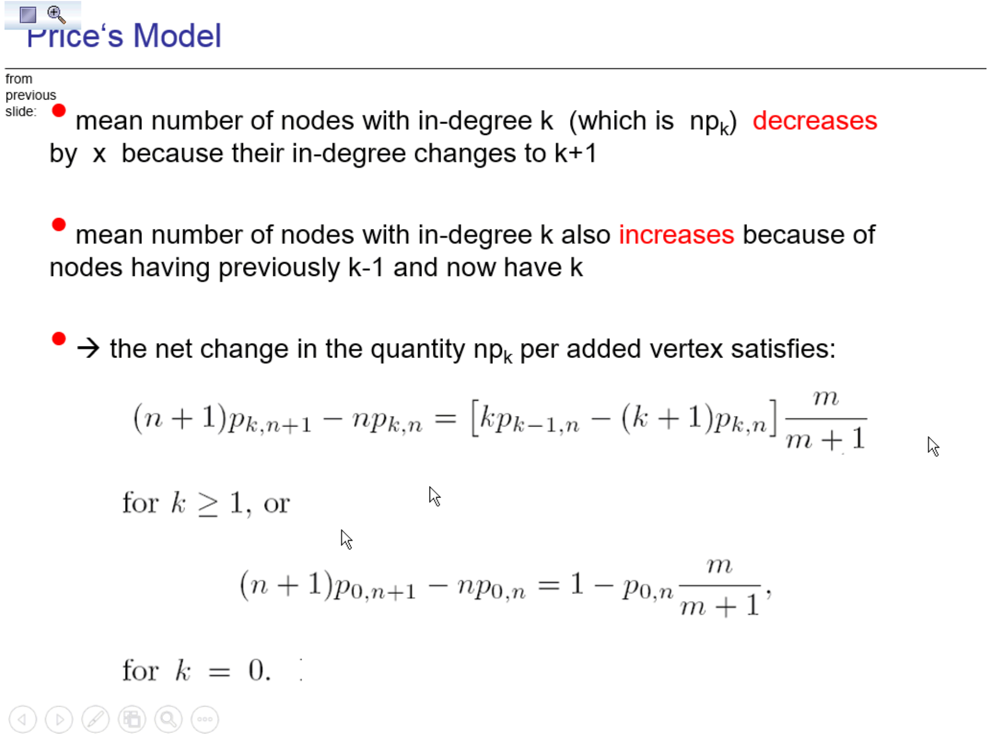
- → the net change in the quantity np_k per added vertex satisfies:

$$(n+1)p_{k,n+1} - np_{k,n} = \left[kp_{k-1,n} - (k+1)p_{k,n}\right]\frac{m}{m+1}$$

for $k \geq 1$, or

$$(n+1)p_{0,n+1} - np_{0,n} = 1 - p_{0,n}\frac{m}{m+1},$$

for $k = 0$.



Price‘s Model

from previous slide:

- mean number of nodes with in-degree k (which is np_k) decreases by x because their in-degree changes to k+1

- mean number of nodes with in-degree k also increases because of nodes having previously k-1 and now have k

- → the net change in the quantity np_k per added vertex satisfies:

$$(n+1)p_{k,n+1} - np_{k,n} = \left[kp_{k-1,n} - (k+1)p_{k,n}\right]\frac{m}{m+1}$$

for $k \geq 1$, or

$$(n+1)p_{0,n+1} - np_{0,n} = 1 - p_{0,n}\frac{m}{m+1},$$

for $k = 0$.

Price's Model

- Computing stationary solutions $p_{k,n+1} = p_{k,n} = p_k$ of this equation we find:

$$p_k \sim k^{-(2+1/m)} \qquad \text{for } n \to \infty$$

- → the desired power law distribution
- we see: „the rich get richer" → power law

Price's Model

- Computing stationary solutions $p_{k,n+1} = p_{k,n} = p_k$ of this equation we find:

$$p_k \sim k^{-(2+1/m)} \qquad \text{for } n \to \infty$$

- → the desired power law distribution
- we see: „the rich get richer" → power law

Barabasi-Albert Model

- same principles as Price's but use undirected edges, intended as model for the WWW
- nodes with fixed degree m are added to the network at each iteration
- edges connect to nodes with probability proportional to current degree of node
- → analogous analysis as for Price's leads to

$$p_k \sim k^{-3} \qquad \text{for } n \to \infty$$

Barabasi-Albert Model and Price's Model

- crucial: linear preferential attachment
- found in a number of real world NW (e.g. citation NW)
- Barabasi-Albert: undirected (not like WWW)
- directed version of Barabasi Albert: attachment prop to sum of out and in- degree: not realistic for e.g. the WWW but for social NW?!
- Price: generates directed acyclic graph: not realistic for SN and WWW
- out-degree of WWW: power-law, Price + BA: constant

Barabasi-Albert Model and Price‘s Model

- crucial: linear preferential attachment
- found in a number of real world NW (e.g. citation NW)
- Barabasi-Albert: undirected (not like WWW)
- directed version of Barabasi Albert: attachment prop to sum of out and in- degree: not realistic for e.g. the WWW but for social NW?!
- Price: generates directed acyclic graph: not realistic for SN and WWW
- out-degree of WWW: power-law, Price + BA: constant

Barabasi-Albert Model and Price‘s Model

- crucial: linear preferential attachment
- found in a number of real world NW (e.g. citation NW)
- Barabasi-Albert: undirected (not like WWW)
- directed version of Barabasi Albert: attachment prop to sum of out and in- degree: not realistic for e.g. the WWW but for social NW?!
- Price: generates directed acyclic graph: not realistic for SN and WWW
- out-degree of WWW: power-law, Price + BA: constant

Barabasi-Albert Model and Price‘s Model

- crucial: linear preferential attachment
- found in a number of real world NW (e.g. citation NW)
- Barabasi-Albert: undirected (not like WWW)
- directed version of Barabasi Albert: attachment prop to sum of out and in- degree: not realistic for e.g. the WWW but for social NW?!
- Price: generates directed acyclic graph: not realistic for SN and WWW
- out-degree of WWW: power-law, Price + BA: constant

Processes on Networks: Percolation

- Assume structure of NW known: what about processes on networks (e.g. spread of info in SN)?
- Percolation: Randomly assign states „occupied“ and „not occupied“ to either edges or vertices → investigate occupied and un-occupied „parts“ separately
- Similarly: Take out nodes / edges, ask for network resilience. E.g. measure resilience via connectednes (e.g. existence of giant component)
- Example: configuration random graph model with power law degree distribution $p_k \sim k^{-\alpha}$; investigate phase transition to / from existing giant component when „occupying“ nodes

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q be the constant fraction of occupied („functional“ / „working“) vertices
- → for vertex with degree k: fraction of occupied neighbors:

$$p(k'|k)=\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → probability that any node is connected to k´ occupied nodes is

$$p_{k'}=p(k')=\sum_k p(k'|k)\,p(k)=\sum_k p(k'|k)\,p_k=\sum_{k=k'}^{\infty} p_k\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → (analysis similar to slide 29 / 30) → for α ≤ 3 : independent of positive q: giant component always exists → random „removal“ of (1-q) nodes leaves NW „unimpressed“

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q be the constant fraction of occupied („functional“ / „working“) vertices
- → for vertex with degree k: fraction of occupied neighbors:

$$p(k'|k)=\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → probability that any node is connected to k´ occupied nodes is

$$p_{k'}=p(k')=\sum_k p(k'|k)\,p(k)=\sum_k p(k'|k)\,p_k=\sum_{k=k'}^{\infty} p_k\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → (analysis similar to slide 29 / 30) → for α ≤ 3 : independent of positive q: giant component always exists → random „removal“ of (1-q) nodes leaves NW „unimpressed“

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q be the constant fraction of occupied („functional“ / „working“) vertices
- → for vertex with degree k: fraction of occupied neighbors:

$$p(k'|k)=\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → probability that any node is connected to k´ occupied nodes is

$$p_{k'}=p(k')=\sum_k p(k'|k)\,p(k)=\sum_k p(k'|k)\,p_k=\sum_{k=k'}^{\infty} p_k\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → (analysis similar to slide 29 / 30) → for α ≤ 3 : independent of positive q: giant component always exists → random „removal“ of (1-q) nodes leaves NW „unimpressed“

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q be the constant fraction of occupied („functional“ / „working“) vertices
- → for vertex with degree k: fraction of occupied neighbors:

$$p(k'|k)=\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → probability that any node is connected to k´ occupied nodes is

$$p_{k'}=p(k')=\sum_k p(k'|k)\,p(k)=\sum_k p(k'|k)\,p_k=\sum_{k=k'}^{\infty} p_k\binom{k}{k'}q^{k'}(1-q)^{k-k'}$$

- → (analysis similar to slide 29 / 30) → for α ≤ 3 : independent of positive q: giant component always exists → random „removal“ of (1-q) nodes leaves NW „unimpressed“

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q be the constant fraction of occupied („functional“ / „working“) vertices

- → for vertex with degree k: fraction of occupied neighbors:

$$p(k'|k) = \binom{k}{k'} q^{k'} (1-q)^{k-k'}$$

- → probability that any node is connected to k´ occupied nodes is

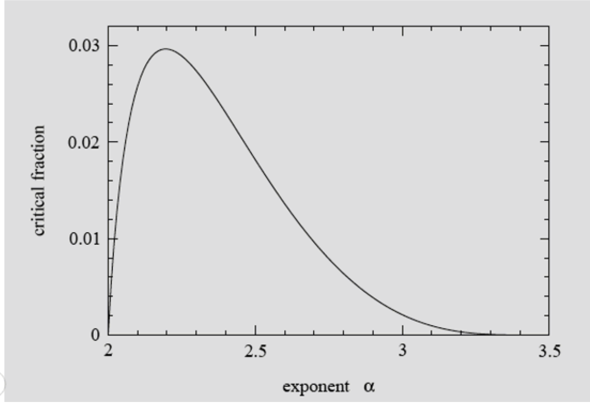
$$p_{k'} = p(k') = \sum_k p(k'|k)\, p(k) = \sum_k p(k'|k)\, pk = \sum_{k=k'}^{\infty} p_k \binom{k}{k'} q^{k'} (1-q)^{k-k'}$$

- → (analysis similar to slide 29 / 30) → for $\alpha \le 3$: independent of positive q: giant component always exists → random „removal“ of (1-q) nodes leaves NW „unimpressed“

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q_k be the fraction of occupied vertices dependent on k (e.g. remove / occupy only high degree nodes)

- → (analysis) → only a small fraction of the high degree nodes needs to be removed to destroy the giant component

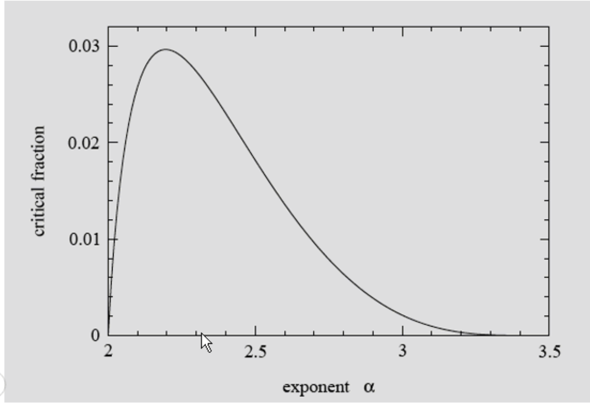


[1]

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q_k be the fraction of occupied vertices dependent on k (e.g. remove / occupy only high degree nodes)

- → (analysis) → only a small fraction of the high degree nodes needs to be removed to destroy the giant component

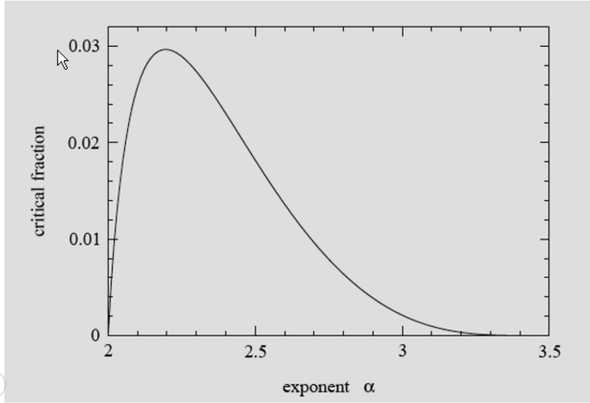


[1]

Processes on Networks: Percolation

- degree distr.: $p_k \sim k^{-\alpha}$;
- let q_k be the fraction of occupied vertices dependent on k (e.g. remove / occupy only high degree nodes)

- → (analysis) → only a small fraction of the high degree nodes needs to be removed to destroy the giant component



[1]

Processes on Networks: Epidemiology

- disease: nodes V = susceptible s $\uplus$ infective i $\uplus$ recovered r
- susceptibles: can be infected;
  infective: have the disease and are contageous,
  recovered: have had the disease and are immune (or dead)

- infection probability / rate β , recovering probability γ
- → SIR model („fully mixed“):

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -\beta i s, \qquad \frac{\mathrm{d}i}{\mathrm{d}t} = \beta i s - \gamma i, \qquad \frac{\mathrm{d}r}{\mathrm{d}t} = \gamma i$$

Processes on Networks: Epidemiology

- disease: nodes V = susceptible s $\uplus$ infective i $\uplus$ recovered r
- susceptibles: can be infected;
  infective: have the disease and are contageous,
  recovered: have had the disease and are immune (or dead)

- infection probability / rate β , recovering probability γ
- → SIR model („fully mixed“):

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -\beta i s, \qquad \frac{\mathrm{d}i}{\mathrm{d}t} = \beta i s - \gamma i, \qquad \frac{\mathrm{d}r}{\mathrm{d}t} = \gamma i$$

Processes on Networks: Epidemiology

- disease: nodes V = susceptible s $\uplus$ infective i $\uplus$ recovered r
- susceptibles: can be infected;
  infective: have the disease and are contageous,
  recovered: have had the disease and are immune (or dead)

- infection probability / rate β , recovering probability γ
- → SIR model („fully mixed“):

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -\beta i s, \qquad \frac{\mathrm{d}i}{\mathrm{d}t} = \beta i s - \gamma i, \qquad \frac{\mathrm{d}r}{\mathrm{d}t} = \gamma i$$

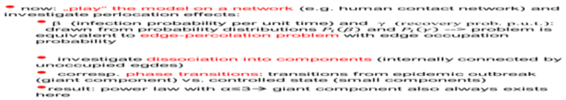
Processes on Networks: Epidemiology

- disease: nodes V = susceptible s $\uplus$ infective i $\uplus$ recovered r
- susceptibles: can be infected;
  infective: have the disease and are contageous,
  recovered: have had the disease and are immune (or dead)

- infection probability / rate β , recovering probability γ
- → SIR model („fully mixed“):

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -\beta i s, \qquad \frac{\mathrm{d}i}{\mathrm{d}t} = \beta i s - \gamma i, \qquad \frac{\mathrm{d}r}{\mathrm{d}t} = \gamma i$$

Processes on Networks: Epidemiology

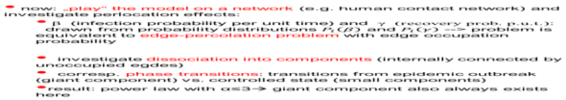


- now: „play“ the model on a network (e.g. human contact network) and investigate perlocation effects:
  - β (infection probability per unit time) and γ (recovery prob. p.u.t.): drawn from probability distributions $P_i(\beta)$ and $P_i(\gamma)$ --> problem is equivalent to edge-percolation problem with edge occupation probability

$$T = 1 - \int_0^\infty P_i(\beta)P_r(\gamma)\,\mathrm{e}^{-\beta/\gamma}\,\mathrm{d}\beta\,\mathrm{d}\gamma.$$

  - investigate dissociation into components (internally connected by unoccupied egdes)
  - corresp. phase transitions: transitions from epidemic outbreak (giant component) vs. controlled state (small components)
  - result: power law with α≤3→ giant component also always exists here

Processes on Networks: Epidemiology

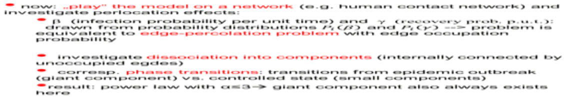


- now: „play“ the model on a network (e.g. human contact network) and investigate perlocation effects:
  - β (infection probability per unit time) and γ (recovery prob. p.u.t.): drawn from probability distributions $P_i(\beta)$ and $P_i(\gamma)$ --> problem is equivalent to edge-percolation problem with edge occupation probability

$$T = 1 - \int_0^\infty P_i(\beta)P_r(\gamma)\,\mathrm{e}^{-\beta/\gamma}\,\mathrm{d}\beta\,\mathrm{d}\gamma.$$

  - investigate dissociation into components (internally connected by unoccupied egdes)
  - corresp. phase transitions: transitions from epidemic outbreak (giant component) vs. controlled state (small components)
  - result: power law with α≤3→ giant component also always exists here

Processes on Networks: Epidemiology



- now: „play“ the model on a network (e.g. human contact network) and investigate perlocation effects:
  - β (infection probability per unit time) and γ (recovery prob. p.u.t.): drawn from probability distributions $P_i(\beta)$ and $P_i(\gamma)$ --> problem is equivalent to edge-percolation problem with edge occupation probability

$$T = 1 - \int_0^\infty P_i(\beta)P_r(\gamma)\,\mathrm{e}^{-\beta/\gamma}\,\mathrm{d}\beta\,\mathrm{d}\gamma.$$

  - investigate dissociation into components (internally connected by unoccupied egdes)
  - corresp. phase transitions: transitions from epidemic outbreak (giant component) vs. controlled state (small components)
  - result: power law with α≤3→ giant component also always exists here

Processes on Networks: Searching and Navigating

- We have seen: Feedback/Eigenvector-Centrality / Page Rank: weight of vertex i (neglecting heuristic corrections):

$$x_i = \lambda^{-1}\textstyle\sum_j A_{ij}x_j \text{ for some } \lambda > 0 \quad \rightarrow \quad \mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

- instead of only looking at in-degrees also look at high out-degree
- node with high in-degree „from“ highly (out-degree-)weighted nodes == „Authority“ :
- node with high out-degree „to“ highly (in-degree-)weighted nodes == „Hub“)
- in-degree based weights: x; out-degree-based weights y

$$\mathbf{A}\mathbf{y} = \lambda\mathbf{x}, \qquad \mathbf{A}^T\mathbf{x} = \mu\mathbf{y} \qquad \rightarrow \qquad \mathbf{A}\mathbf{A}^T\mathbf{x} = \lambda\mu\mathbf{x}$$

Processes on Networks: Searching and Navigating

- We have seen: Feedback/Eigenvector-Centrality / Page Rank: weight of vertex i (neglecting heuristic corrections):

$$x_i = \lambda^{-1} \sum_j A_{ij} x_j \text{ for some } \lambda > 0 \quad \rightarrow \quad \mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

- instead of only looking at in-degrees also look at high out-degree
- node with high in-degree „from“
  highly (out-degree-)weighted nodes == „Authority“ :
- node with high out-degree „to“
  highly (in-degree-)weighted nodes == „Hub“)
- in-degree based weights: x; out-degree-based weights y

$$\mathbf{A}\mathbf{y} = \lambda \mathbf{x}, \qquad \mathbf{A}^T\mathbf{x} = \mu \mathbf{y} \qquad \rightarrow \qquad \mathbf{A}\mathbf{A}^T\mathbf{x} = \lambda\mu\mathbf{x}$$

Processes on Networks: Searching and Navigating

- We have seen: Feedback/Eigenvector-Centrality / Page Rank: weight of vertex i (neglecting heuristic corrections):

$$x_i = \lambda^{-1} \sum_j A_{ij} x_j \text{ for some } \lambda > 0 \quad \rightarrow \quad \mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

- instead of only looking at in-degrees also look at high out-degree
- node with high in-degree „from“
  highly (out-degree-)weighted nodes == „Authority“ :
- node with high out-degree „to“
  highly (in-degree-)weighted nodes == „Hub“)
- in-degree based weights: x; out-degree-based weights y

$$\mathbf{A}\mathbf{y} = \lambda \mathbf{x}, \qquad \mathbf{A}^T\mathbf{x} = \mu \mathbf{y} \qquad \rightarrow \qquad \mathbf{A}\mathbf{A}^T\mathbf{x} = \lambda\mu\mathbf{x}$$

Processes on Networks: Searching and Navigating

- We have seen: Feedback/Eigenvector-Centrality / Page Rank: weight of vertex i (neglecting heuristic corrections):

$$x_i = \lambda^{-1} \sum_j A_{ij} x_j \text{ for some } \lambda > 0 \quad \rightarrow \quad \mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

- instead of only looking at in-degrees also look at high out-degree
- node with high in-degree „from“
  highly (out-degree-)weighted nodes == „Authority“ :
- node with high out-degree „to“
  highly (in-degree-)weighted nodes == „Hub“)
- in-degree based weights: x; out-degree-based weights y

$$\mathbf{A}\mathbf{y} = \lambda \mathbf{x}, \qquad \mathbf{A}^T\mathbf{x} = \mu \mathbf{y} \qquad \rightarrow \qquad \mathbf{A}\mathbf{A}^T\mathbf{x} = \lambda\mu\mathbf{x}$$

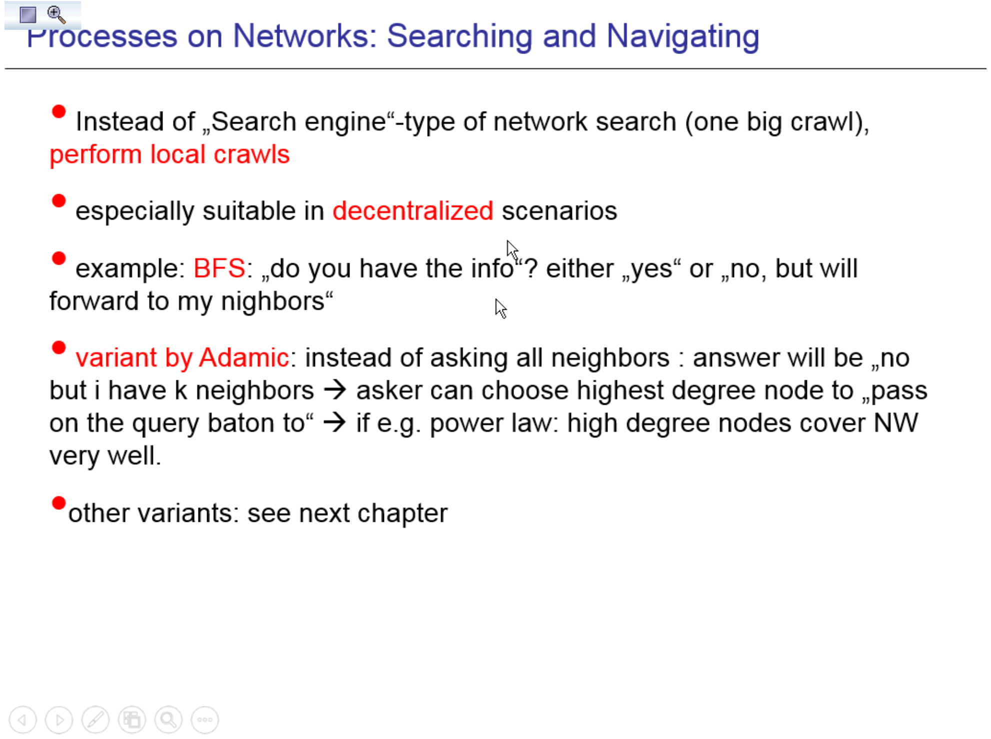
Processes on Networks: Searching and Navigating

- We have seen: Feedback/Eigenvector-Centrality / Page Rank: weight of vertex i (neglecting heuristic corrections):

$$x_i = \lambda^{-1} \sum_j A_{ij} x_j \text{ for some } \lambda > 0 \quad \rightarrow \quad \mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

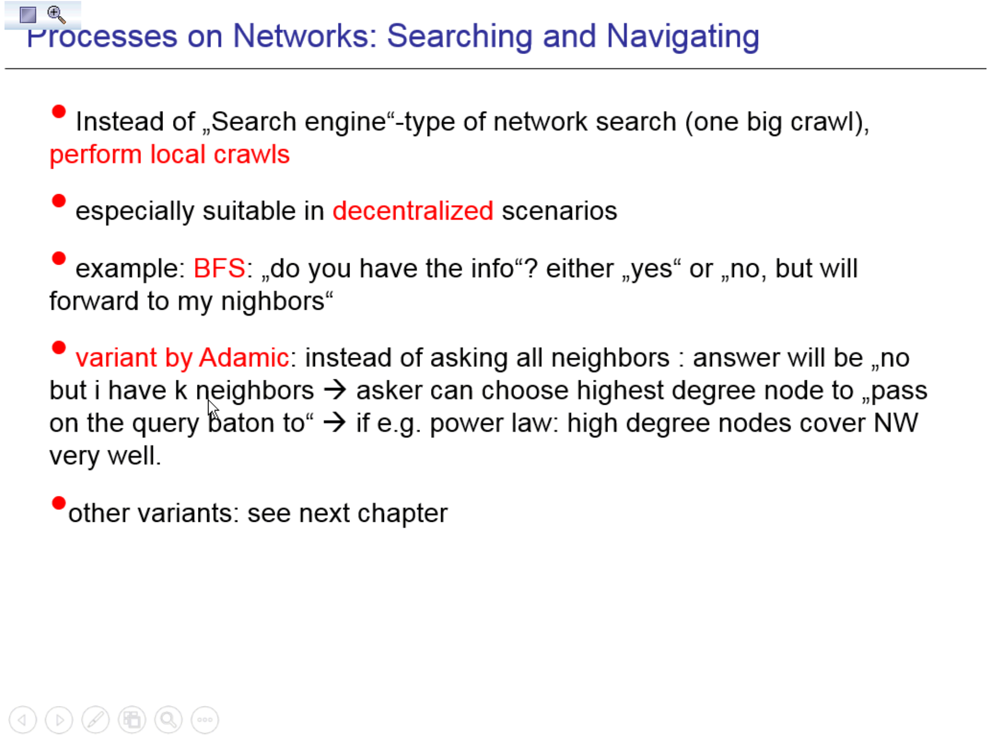
- instead of only looking at in-degrees also look at high out-degree
- node with high in-degree „from“
  highly (out-degree-)weighted nodes == „Authority“ :
- node with high out-degree „to“
  highly (in-degree-)weighted nodes == „Hub“)
- in-degree based weights: x; out-degree-based weights y

$$\mathbf{A}\mathbf{y} = \lambda \mathbf{x}, \qquad \mathbf{A}^T\mathbf{x} = \mu \mathbf{y} \qquad \rightarrow \qquad \mathbf{A}\mathbf{A}^T\mathbf{x} = \lambda\mu\mathbf{x}$$



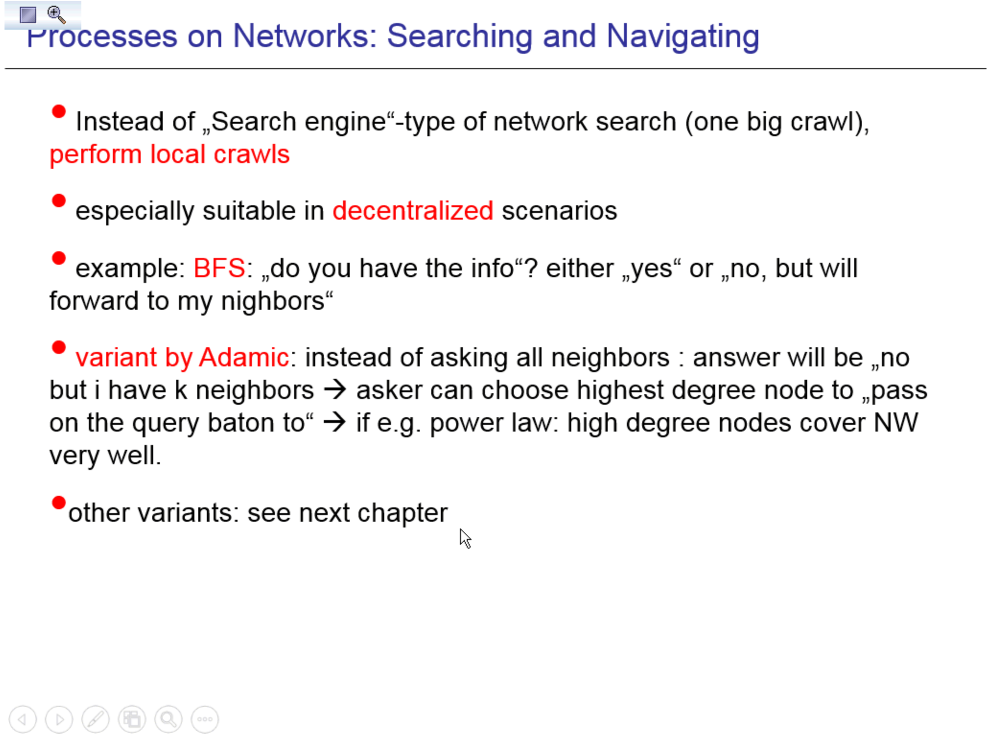
Processes on Networks: Searching and Navigating

- Instead of „Search engine“-type of network search (one big crawl), perform local crawls
- especially suitable in decentralized scenarios
- example: BFS: „do you have the info“? either „yes“ or „no, but will forward to my nighbors“
- variant by Adamic: instead of asking all neighbors : answer will be „no but i have k neighbors → asker can choose highest degree node to „pass on the query baton to“ → if e.g. power law: high degree nodes cover NW very well.
- other variants: see next chapter



Processes on Networks: Searching and Navigating

- Instead of „Search engine“-type of network search (one big crawl), perform local crawls
- especially suitable in decentralized scenarios
- example: BFS: „do you have the info“? either „yes“ or „no, but will forward to my nighbors“
- variant by Adamic: instead of asking all neighbors : answer will be „no but i have k neighbors → asker can choose highest degree node to „pass on the query baton to“ → if e.g. power law: high degree nodes cover NW very well.
- other variants: see next chapter



Processes on Networks: Searching and Navigating

- Instead of „Search engine“-type of network search (one big crawl), perform local crawls
- especially suitable in decentralized scenarios
- example: BFS: „do you have the info“? either „yes“ or „no, but will forward to my nighbors“
- variant by Adamic: instead of asking all neighbors : answer will be „no but i have k neighbors → asker can choose highest degree node to „pass on the query baton to“ → if e.g. power law: high degree nodes cover NW very well.
- other variants: see next chapter