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0.8 Case study: Counting words

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Design principle:

> *Solve problem in a sequence of small steps*
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Unix pipes!

Step 1: Break input into words

```
"never say never again"
          ↓
["never", "say", "never", "again"]
```


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```
"never say never again"
   function ↓ words
["never", "say", "never", "again"]
```

Predefined in Prelude

Step 2: Sort words

```
["never", "say", "never", "again"]
```

$\downarrow$

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["again", "never", "never", "say"]
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Step 2: Sort words

```
["never", "say", "never", "again"]
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function $\downarrow$ `sort`

```
["again", "never", "never", "say"]
```

Predefined in `Data.List`

Step 3: Group equal words together

```
["again", "never", "never", "say"]
```

$\downarrow$

```
[["again"], ["never", "never"], ["say"]]
```

Step 3: Group equal words together

```
["again", "never", "never", "say"]
```

function $\downarrow$ `group`

```
[["again"], ["never", "never"], ["say"]]
```

Predefined in `Data.List`

Step 4: Count each group

```
[["again"], ["never", "never"], ["say"]]
        ↓
[("again", 1), ("never", 2), ("say", 1)]
```

Step 4: Count each group

```
[["again"], ["never", "never"], ["say"]]
        ↓ map (\ws -> (head ws, length ws))
[("again", 1), ("never", 2), ("say", 1)]
```

Step 5: Format each group

```
[("again", 1), ("never", 2), ("say", 1)]
        ↓
["again: 1", "never: 2", "say: 1"]
```

Step 5: Format each group

```
[("again", 1), ("never", 2), ("say", 1)]
        ↓ map (\(w,n) -> (w ++ ": " ++ show n))
["again: 1", "never: 2", "say: 1"]
```

Step 6: Combine the lines

```
["again: 1", "never: 2", "say: 1"]
```

↓

```
"again: 1\nnever: 2\nsay: 1\n"
```

Step 6: Combine the lines

```
["again: 1", "never: 2", "say: 1"]
```

function ↓ `unlines`

```
"again: 1\nnever: 2\nsay: 1\n"
```

Predefined in Prelude

The solution

```
countWords :: String -> String
countWords  =
  unlines
  . map (\(w,n) -> w ++ ": " ++ show n)
  . map (\ws -> (head ws, length ws))
  . group
  . sort
  . words
```

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countWords :: String -> String
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Merging maps

Can we merge two consecutive `maps`?

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map f . map g =
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The optimized solution

```
countWords :: String -> String
countWords  =
  unlines
  . map (\ws -> head ws ++ ": " ++ show(length ws))
  . group
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Proving `map f . map g = map (f.g)`

First we prove (why?)

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map f (map g xs) = map (f.g) xs
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by induction on `xs`:

- Base case:
```
map f (map g []) = []
map (f.g) [] = []
```
- Induction step:
```
map f (map g (x:xs))
= f (g x) : map f (map g xs)
= f (g x) : map (f.g) xs       -- by IH
map (f.g) (x:xs)
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$\Longrightarrow$ `(map f . map g) xs = map f (map g xs) = map (f.g) xs`

$\Longrightarrow$ `(map f . map g) = map (f.g)` by extensionality

7. Type Classes

Remember: type classes enable overloading

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Example

```
elem ::
elem x  =  any (== x)
```

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Example

```
elem ::  Eq a => a -> [a] -> Bool
elem x  =  any (== x)
```

where Eq is the class of all types with ==

In general:

Type classes are collections of types
that implement some fixed set of functions

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Example

```
class Eq a where
  (==) :: a -> a -> Bool
```

Note: the type of `(==)` outside the class context is
`Eq a => a -> a -> Bool`

The general form of a class declaration:

```
class C a where
  f1 :: T1
  ...
  fn :: Tn
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Code that works not just for one type
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Instance

A type T is an instance of a class C
if T supports all the functions of C.

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Code that works not just for one type
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Warning Terminology clash:
Type T_1 is an *instance* of type T_2
if T_1 is the result of replacing type variables in T_2.
For example `(Bool,Int)` is an instance of `(a,b)`.

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Example

```
instance Eq Bool where
  True  == True   =  True
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Instances can be constrained:

Example

```
instance Eq a => Eq [a] where
  []     == []     =  True
  (x:xs) == (y:ys) =  x == y && xs == ys
  _      == _      =  False
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instance Eq a => Eq [a] where
  []     == []     =  True
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```


Possibly with multiple constraints:

Example

```
instance (Eq a, Eq b) => Eq (a,b) where
  (x1,y1) == (x2,y2)  =  x1 == x2 && y1 == y2
```

The general form of the **instance** statement:

instance (*context*) **=>** *C* *T* **where**
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instance (*context*) **=>** *C* *T* **where**
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T	is a type
context	is a list of assumptions $C_i\ T_i$
definitions	are definitions of the functions of class *C*

Subclasses

Example

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class Eq a => Ord a where
  (<=), (<) :: a -> a -> Bool
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Because `Bool` is already an instance of `Eq`,
we can now make it an instance of `Ord`:

Subclasses

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class Eq a => Ord a where
  (<=), (<) :: a -> a -> Bool
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Class `Ord` inherits all the operations of class `Eq`

Because `Bool` is already an instance of `Eq`,
we can now make it an instance of `Ord`:

```
instance Ord Bool where
  b1 <= b2  =  not b1 || b2
  b1 < b2   =  b1 <= b2 && not(b1 == b2)
```

From the Prelude: Eq, Ord, Show

```
class Eq a where
  (==), (/=) :: a -> a -> Bool
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class Eq a => Ord a where
  (<=), (<), (>=), (>) :: a -> a -> Bool
  -- default definitions:
  x < y  =  x <= y && x /= y
  x > y  =  y < x
  x >= y =  y <= x
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class Show a where
  show :: a -> String
```

From the Prelude: Eq, Ord, Show

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8. Algebraic `data` Types

So far: no really new types,
just compositions of existing types

Example: `type String = [Char]`

Now: `data` defines *new* types

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Introduction by example: From enumerated types

8.1 `data` by example

Bool

From the Prelude:

```
data Bool = False | True
```

Bool

From the Prelude:

```
data Bool = False | True

not :: Bool -> Bool
not False  =  True
not True   =  False
```

Bool

From the Prelude:

```haskell
data Bool = False | True

not :: Bool -> Bool
not False  =  True
not True   =  False

(&&) :: Bool -> Bool -> Bool
False && q  =  False
True  && q  =  q
```

deriving

Bool

From the Prelude:

```haskell
data Bool = False | True

not :: Bool -> Bool
not False  =  True
not True   =  False

(&&) :: Bool -> Bool -> Bool
False && q  =  False
True  && q  =  q
```


```haskell
(||) :: Bool -> Bool -> Bool
False || q  = q
True  || q  = True
```

deriving

```haskell
instance Eq Bool where
  True  == True   =  True
  False == False  =  True
  _     == _      =  False
```

deriving

```
instance Eq Bool where
  True  == True   =  True
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instance Show Bool where
  show True   =  "True"
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  show True   =  "True"
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Better: let Haskell write the code for you:

```
data Bool = False | True
            deriving (Eq, Show)
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deriving

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instance Eq Bool where
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  show True   =  "True"
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Better: let Haskell write the code for you:

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data Bool = False | True
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`deriving` supports many more classes: `Ord`, `Read`, ...

Warning

Do not forget to make your `data` types instances of `Show`

Otherwise Haskell cannot even print values of your type

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QuickCheck does not automatically work for `data` types

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Warning

QuickCheck does not automatically work for `data` types

You have to write your own test data generator. Later.

Season

```
data Season = Spring | Summer | Autumn | Winter
              deriving (Eq, Show)
```

Shape

```
type Radius = Float
type Width  = Float
type Height = Float
```

Season

```
data Season = Spring | Summer | Autumn | Winter
              deriving (Eq, Show)

next :: Season -> Season
next Spring = Summer
next Summer = Autumn
next Autumn = Winter
next Winter = Spring
```

Shape

```
type Radius = Float
type Width  = Float
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Shape

```
type Radius = Float
type Width  = Float
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data Shape = Circle Radius | Rect Width Height
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             deriving (Eq, Show)
```

Some values of type Shape: `Circle 1.0`

Shape

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type Radius = Float
type Width  = Float
type Height = Float

data Shape = Circle Radius | Rect Width Height
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```

Some values of type Shape: `Circle 1.0`
`Rect 0.9 1.1`
`Circle (-2.0)`

Shape

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type Radius = Float
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Some values of type Shape: `Circle 1.0`
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```
area :: Shape -> Float
```

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Some values of type Shape: `Circle 1.0`
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```
area :: Shape -> Float
area (Circle r)  =  pi * r^2
```

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```
area :: Shape -> Float
area (Circle r)  =  pi * r^2
area (Rect w h)  =  w * h
```

Maybe

From the Prelude:

```
data Maybe a = Nothing | Just a
               deriving (Eq, Show)
```

Some values of type Maybe: `Nothing ::    Maybe a`

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Some values of type Maybe:

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Nothing ::    Maybe a
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Just "?" ::
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Some values of type Maybe:

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```
lookup :: Eq a => a -> [(a,b)] -> Maybe b
```

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Some values of type Maybe:

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Nothing ::   Maybe a
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Just "?" ::  Maybe String
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lookup :: Eq a => a -> [(a,b)] -> Maybe b
lookup key []  =
```

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Some values of type Maybe:

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Nothing ::   Maybe a
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Some values of type Maybe:

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```
lookup :: Eq a => a -> [(a,b)] -> Maybe b
lookup key []  =  Nothing
lookup key ((x,y):xys)
  | key == x    =
```

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Some values of type Maybe:

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Nothing ::   Maybe a
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  | otherwise  =  lookup key xys
```

Nat

Natural numbers:

```
data Nat = Zero | Suc Nat
           deriving (Eq, Show)
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Some values of type Nat:

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$\vdots$

```
add :: Nat -> Nat -> Nat
```

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Natural numbers:

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data Nat = Zero | Suc Nat
           deriving (Eq, Show)
```

Some values of type Nat:

```
Zero
Suc Zero
Suc (Suc Zero)
  ⋮
```

```
add :: Nat -> Nat -> Nat
add Zero n  =  n
```

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Some values of type Nat:

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Suc Zero
Suc (Suc Zero)
  ⋮
```

```
add :: Nat -> Nat -> Nat
add Zero n  =  n
add (Suc m) n  =
```

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data Nat = Zero | Suc Nat
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Some values of type Nat:

```
Zero
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  ⋮
```

```
add :: Nat -> Nat -> Nat
add Zero n  =  n
add (Suc m) n  =  Suc (add m n)

mul :: Nat -> Nat -> Nat
```

Lists

From the Prelude:

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data [a] = [] | (:) a [a]
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