

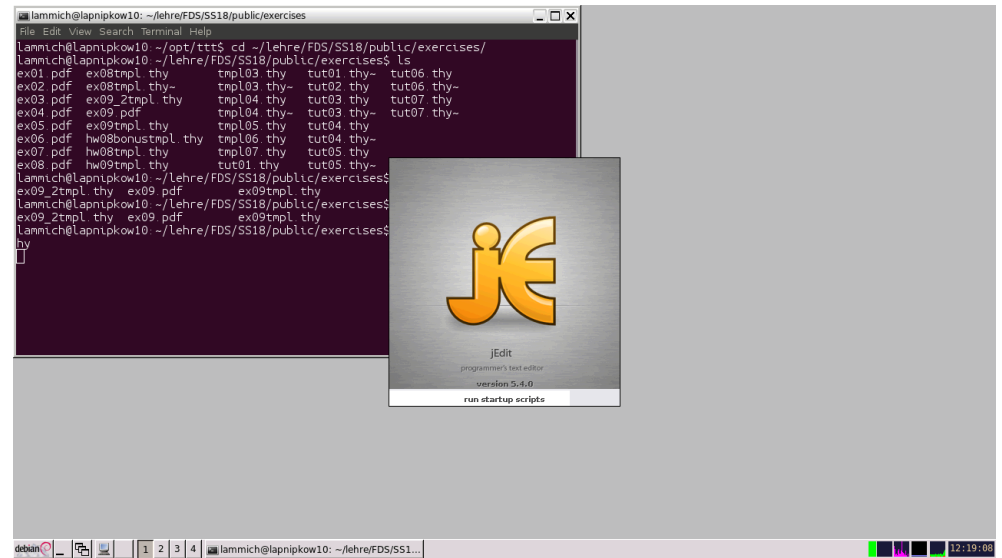
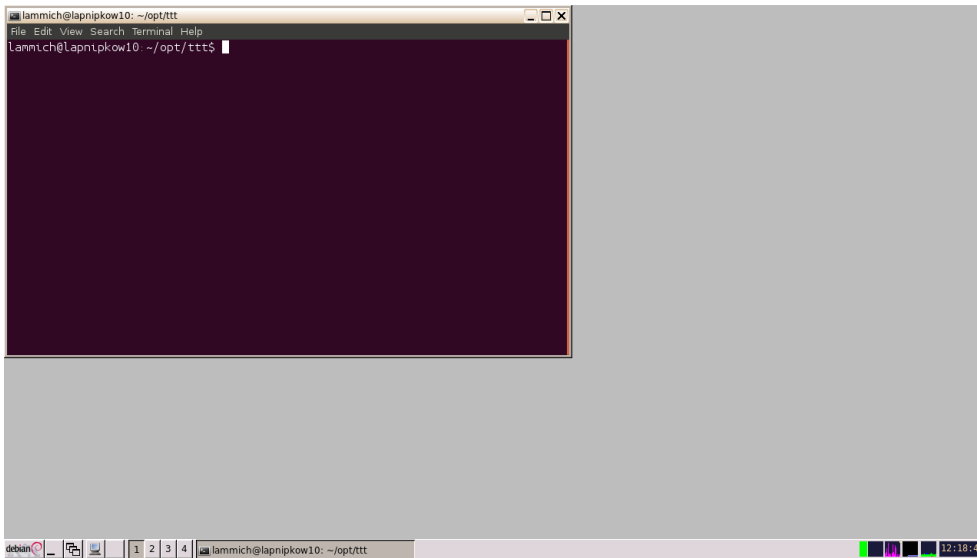
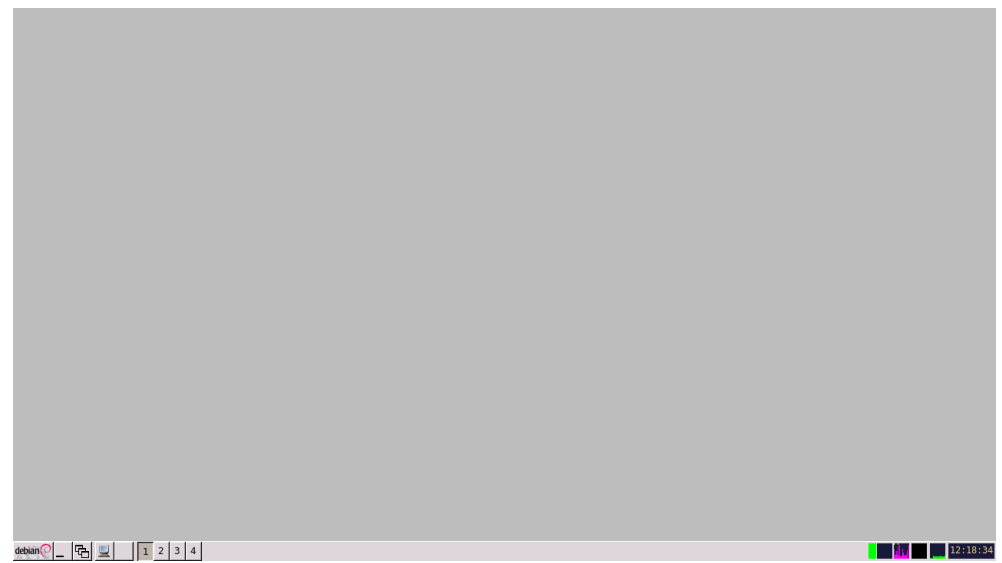
Script generated by TTT

Title: Lammich: FDS Tutorial (08.06.2018)

Date: Fri Jun 08 12:18:34 CEST 2018

Duration: 99:08 min

Pages: 97



```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
1 (*>*)
2
3
4
5
6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' _ _ = undefined"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 sorry
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
11.1 (169/517) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 540/170MB 12:19 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:19:59
```

```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
1 (*>*)
2
3
4
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6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' _ _ = undefined"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 sorry
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
21.1 (49/517) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 593/170MB 12:21 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:21:07
```

```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
1 (*>*)
2
3
4
5
6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' _ _ = undefined"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 sorry
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
20.66 (480/517) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 700/170MB 12:21 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:21:33
```

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Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
1 (*>*)
2
3
4
5
6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' _ _ = undefined"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 sorry
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
20.58 (472/517) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 720/170MB 12:22 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:22:45
```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
4 (*>*)
5
6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' _ _ = undefined"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 sorry
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
20.50 (464/517) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 780/11/0MB 12:23 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
4 (*>*)
5
6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' _ _ = undefined"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 sorry
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
17.6 (390/517) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 775/11/0MB 12:23 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
4 (*>*)
5
6
7
8 text {* \ExerciseSheet{9}{8.-6.-2018} *}
9
10 text <\Exercise{Indicate Unchanged by Option}
11
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' x t = Some (ins x t)"
19
20 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
21 by auto
22 (*<*)
23 end
24 (*>*)
25
Output Query Sledgehammer Symbols
18.10 (400/524) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 91/11/71MB 12:23 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' x Leaf = Some Leaf x Leaf" |
19 "ins' x (B l a r) =
20 (case cmp x a of
21 LT => balil (ins' x l) a r |
22 GT => balir l a (ins' x r) |
23 EQ => B l a r) |
24 "ins x (R l a r) =
25 (case cmp x a of
26 LT => R (ins' x l) a r |
27 GT => R l a (ins' x r) |
28 EQ => R l a r)"
29
30 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
31 by auto
32 (*<*)
33 end
34
Output Query Sledgehammer Symbols
18.20 (410/772) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 64/1/71MB 12:25 PM

```

```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' x Leaf = Some (R Leaf x Leaf)" |
19 "ins' x (B l a r) =
20 (case cmp x a of
21 LT => balLl (ins' x l) a r |
22 GT => balRl l a (ins' x r) |
23 EQ => B l a r)" |
24 "ins x (R l a r) =
25 (case cmp x a of
26 LT => R (ins' x l) a r |
27 GT => R l a (ins' x r) |
28 EQ => R l a r)"
29
30 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
31 by auto
32 (*<*)
33 end
Output Query Sledgehammer Symbols
21.11 (479/775) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 750/1171MB 12:26 PM
debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:26:09
```

```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' x Leaf = Some (R Leaf x Leaf)" |
19 "ins' x (B l a r) =
20 (case cmp x a of
21 LT => balLl (ins' x l) a r |
22 GT => balRl l a (ins' x r) |
23 EQ => B l a r)" |
24 "ins x (R l a r) =
25 (case cmp x a of
26 LT => R (ins' x l) a r |
27 GT => R l a (ins' x r) |
28 EQ => R l a r)"
29
30 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
31 by auto
32 (*<*)
33 end
Output Query Sledgehammer Symbols
21.18 (486/775) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 750/1171MB 12:26 PM
debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:26:15
```

```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' x Leaf = Some (R Leaf x Leaf)" |
19 "ins' x (B l a r) =
20 (case cmp x a of
21 LT => balLl (ins' x l) a r |
22 GT => balRl l a (ins' x r) |
23 EQ => B l a r)" |
24 "ins x (R l a r) =
25 (case cmp x a of
26 LT => R (ins' x l) a r |
27 GT => R l a (ins' x r) |
28 EQ => R l a r)"
29
30 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
31 by auto
32 (*<*)
33 end
Output Query Sledgehammer Symbols
21.11 (479/775) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 67/1269MB 12:27 PM
debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:27:53
```

```
Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
17 where
18 "ins' x Leaf = Some (R Leaf x Leaf)" |
19 "ins' x (B l a r) =
20 (case cmp x a of
21 LT => balLl (ins' x l) a r |
22 GT => balRl l a (ins' x r) |
23 EQ => B l a r)" |
24 "ins x (R l a r) =
25 (case cmp x a of
26 LT => R (ins' x l) a r |
27 GT => R l a (ins' x r) |
28 EQ => R l a r)"
29
30 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
31 by auto
32 (*<*)
33 end
Output Query Sledgehammer Symbols
21.14 (482/775) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 81/1269MB 12:28 PM
debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:28:10
```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
16 where
17 "ins' x Leaf = Some (R Leaf x Leaf)" |
18 "ins' x (B l a r) =
19 (case cmp x a of
20 LT => balil (ins' x l) a r |
21 GT => balir l a (ins' x r) |
22 EQ => B l a r)" |
23 "ins x (R l a r) =
24 (case cmp x a of
25 LT => R (ins' x l) a r |
26 GT => R l a (ins' x r) |
27 EQ => R l a r)"
28
29 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
30 by auto
31 (*<*)
32 end
33
Output Query Sledgehammer Symbols
17.6 (390/775) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 117/1269MB 12:28 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
12 Write an insert function for red-black trees that either inserts the element
13 and returns a new tree, or returns None if the element was already in the tree
14 >
15
16 term map_option
17
18 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
19 where
20 "ins' x Leaf = Some (R Leaf x Leaf)" |
21 "ins' x (B l a r) =
22 (case cmp x a of
23 LT => balil (ins' x l) a r |
24
25 "map_option"
26 :: "('a => 'b) => 'a option => 'b option"
27
28 Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
19.5 (406/792) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 339/1269MB 12:29 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
16 term map_option
17
18 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
19 where
20 "ins' x Leaf = Some (R Leaf x Leaf)" |
21 "ins' x (B l a r) =
22 (case cmp x a of
23 LT => balil (ins' x l) a r |
24 GT => balir l a (ins' x r) |
25 EQ => B l a r)" |
26 "ins x (R l a r) =
27 (case cmp x a of
28
Type unification failed: Clash of types "_ option" and "(_, _) tree"
Type error in application: incompatible operand type
Operator: balil :: (??'a, color) tree => ??'a => (??'a, color) tree => (??'a, color) tree
Operand: ins' x l :: ('a, color) tree option
Output Query Sledgehammer Symbols
23.11 (496/792) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 2/1269MB 12:30 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
16 term map_option
17
18 fun ins' :: "'a::linorder => 'a rbt => 'a rbt option"
19 where
20 "ins' x Leaf = Some (R Leaf x Leaf)" |
21 "ins' x (B l a r) =
22 (case cmp x a of
23 LT => casbalil (ins' x l) a r |
24 GT => balir l a (ins' x r) |
25 EQ => B l a r)" |
26 "ins x (R l a r) =
27 (case cmp x a of
28
Type unification failed: Clash of types "_ option" and "(_, _) tree"
Type error in application: incompatible operand type
Operator: balil :: (??'a, color) tree => ??'a => (??'a, color) tree => (??'a, color) tree
Operand: ins' x l :: ('a, color) tree option
Output Query Sledgehammer Symbols
23.15 (500/796) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 2/1269MB 12:30 PM

```

Isabelle2017 - ex09tmplthy

```

16 term map_option
17
18 fun ins :: "'a::linorder ⇒ 'a rbt ⇒ 'a rbt option"
19 where
20 "ins' x Leaf = Some (R Leaf x Leaf)" |
21 "ins' x (B l a r) =
22 (case cmp x a of
23 LT ⇒ (case ins' x l of None ⇒ None | Some l' ⇒ Some (bali l' a r)) |
24 GT ⇒ baliR l a (ins' x r) |
25 EQ ⇒ B l a r)" |
26 "ins x (R l a r) =
27 (case cmp x a of

```

Type unification failed: Clash of types "_ option" and "(_, _) tree"

Type error in application: incompatible operand type

Operator: baliR l a :: ('a, color) tree ⇒ ('a, color) tree
Operand: ins' x r :: ('a, color) tree option

23.72 (657/834) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 6871:69MB 12:31 PM

Isabelle2017 - ex09tmplthy

```

18 fun ins :: "'a::linorder ⇒ 'a rbt ⇒ 'a rbt option"
19 where
20 "ins' x Leaf = Some (R Leaf x Leaf)" |
21 "ins' x (B l a r) =
22 (case cmp x a of
23 LT ⇒ (case ins' x l of None ⇒ None | Some l' ⇒ Some (bali l' a r)) |
24 GT ⇒ (case ins' x r of None ⇒ None | Some r' ⇒ Some (baliR l a r')) |
25 EQ ⇒ B l a r)" |
26 "ins x (R l a r) =
27 (case cmp x a of
28 LT ⇒ R (ins' x l) a r |
29 GT ⇒ R l a (ins' x r) |

```

Type unification failed: Clash of types "(_, _) tree" and "_ option"

Type error in application: incompatible operand type

Operator: case_cons (case_elem GT (case ins' x r of None ⇒ None | Some r' ⇒ Some (baliR l a r'))) :: (cmp_val ⇒ ('a, color) tree option) ⇒ cmp_val ⇒ ('a, color) tree option
Operand: case_cons (case_elem EQ (B l a r)) case_nil :: cmp_val ⇒ ('a, color) tree

25.18 (653/876) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 2071:269MB 12:32 PM

Isabelle2017 - ex09tmplthy

```

25 EQ ⇒ None)" |
26 "ins x (R l a r) =
27 (case cmp x a of
28 LT ⇒ (case ins' x l of None ⇒ None | Some l' ⇒ Some (R l' a r)) |
29 GT ⇒ (case ins' x r of None ⇒ None | Some r' ⇒ Some (R l a r')) |
30 EQ ⇒ None)"
31
32 lemma "invc t ⇒ case ins' x t of None ⇒ ins x t = t | Some t' ⇒ ins x t = t'"
33
34 (*<*)
35 end

```

Type unification failed: Clash of types "_ option" and "(_, _) tree"

Type error in application: incompatible operand type

Operator: op = (ins x (R l a r)) :: ('a, color) tree ⇒ bool
Operand: case cmp x a of LT ⇒ case ins' x l of None ⇒ None | Some l' ⇒ Some (R l' a r) | GT ⇒ case ins' x r of None ⇒ None | Some r' ⇒ Some (R l a r') | EQ ⇒ None ::

31.1 (849/945) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 741:247MB 12:34 PM

Isabelle2017 - ex09tmplthy

```

26 "ins x (R l a r) =
27 (case cmp x a of
28 LT ⇒ (case ins' x l of None ⇒ None | Some l' ⇒ Some (R l' a r)) |
29 GT ⇒ (case ins' x r of None ⇒ None | Some r' ⇒ Some (R l a r')) |
30 EQ ⇒ None)"
31
32 lemma "invc t ⇒ case ins' x t of None ⇒ ins x t = t | Some t' ⇒ ins x t = t'"
33
34 (*<*)
35 end
36 (*<*)

```

Type unification failed: Clash of types "_ option" and "(_, _) tree"

Type error in application: incompatible operand type

Operator: op = (ins x (R l a r)) :: ('a, color) tree ⇒ bool
Operand: case cmp x a of LT ⇒ case ins' x l of None ⇒ None | Some l' ⇒ Some (R l' a r) | GT ⇒ case ins' x r of None ⇒ None | Some r' ⇒ Some (R l a r') | EQ ⇒ None ::

31.1 (849/945) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 391:247MB 12:35 PM

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
28 LT => (case ins' x l of None => None | Some l' => Some (R l' a r)) |
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None) "
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33
34 (*<*)
35 end
36 (*>*)
37
proof (prove)
goal (1 subgoal):
1. invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'

```

33.1 (929/946) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 370/1247MB 12:36 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:36:09

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
25 EQ => None) " |
26 "ins' x (R l a r) =
27 (case cmp x a of
28 LT => (case ins' x l of None => None | Some l' => Some (R l' a r)) |
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None) "
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33
34 (*<*)
35 end
36 (*>*)
37
proof (prove)
goal (1 subgoal):
1. invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'

```

33.1 (929/946) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 380/1247MB 12:36 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:36:33

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
25 EQ => None) " |
26 "ins' x (R l a r) =
27 (case cmp x a of
28 LT => (case ins' x l of None => None | Some l' => Some (R l' a r)) |
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None) "
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33
34 (*<*)
35 end
36 (*>*)
37
proof (prove)
goal (1 subgoal):
1. invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'

```

33.3 (931/949) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 447/1247MB 12:38 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy (modified) 12:38:56

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
30 EQ => None) "
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33
34 apply (induction x t rule: ins'.induct)
35 apply auto
36
37 (*<*)
38 end
39 (*>*)
40
proof (prove)
goal (4 subgoals):
1.  $\lambda x l a r.$ 
   [case ins' x r of None => ins x r = r | Some xa => ins x r = xa; invc l; invc r; a < x]
   => case case ins' x r of None => None | Some r' => Some (balR l a r') of
       None => ins x (B l a r) = B l a r | Some xa => ins x (B l a r) = xa
2.  $\lambda x l a r.$ 
   [case ins' x l of None => ins x l = l | Some xa => ins x l = xa; invc l; invc r; x < a]
   => case case ins' x l of None => None | Some l' => Some (balL l' a r) of

```

34.9 (979/1001) (isabelle.isabelle.UTF-8-isabelle)Nm r o UG 891/1247MB 12:39 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex09tmplthy 12:39:42

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
28 LT => (case ins' x l of None => None | Some l' => Some (R l' a r)) |
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None)
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33 apply (induction x t rule: ins'.induct)
34 apply (auto split: option.split)
35
36 (**)
37 end

proof (prove)
goal (3 subgoals):
1.  $\forall x. \text{invc } () \Rightarrow \text{case ins' } x () \text{ of None} \Rightarrow \text{ins } x () = () \mid \text{Some } xa \Rightarrow \text{ins } x () = xa$ 
2.  $\forall x \ l \ a \ r. \begin{cases} [\text{cmp } x \ a = \text{LT}; \text{invc } l] \Rightarrow \text{case ins' } x \ l \text{ of None} \Rightarrow \text{ins } x \ l = l \mid \text{Some } xa \Rightarrow \text{ins } x \ l = xa; \\ [\text{cmp } x \ a = \text{GT}; \text{invc } r] \Rightarrow \text{case ins' } x \ r \text{ of None} \Rightarrow \text{ins } x \ r = r \mid \text{Some } xa \Rightarrow \text{ins } x \ r = xa; \\ \text{invc } (B \ l \ a \ r)] \\ \Rightarrow \text{case ins' } x \ (B \ l \ a \ r) \text{ of None} \Rightarrow \text{ins } x \ (B \ l \ a \ r) = B \ l \ a \ r \mid \text{Some } xa \Rightarrow \text{ins } x \ (B \ l \ a \ r) = xa \end{cases}$ 
3.  $\forall x \ l \ a \ r.$ 

Output Query Sledgehammer Symbols
31.1 (850/1023) (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 24/1.269MB 12:40 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None)
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33 apply (induction x t rule: ins'.induct)
34 apply (auto split: option.split)
35
36 (**)
37 end

proof (prove)
goal (4 subgoals):
1.  $\forall x \ l \ a \ r. \begin{cases} [\text{ins } x \ r = r; \text{invc } l; \text{invc } r; \text{ins' } x \ l = \text{None}; \text{ins' } x \ r = \text{None}; a < x] \Rightarrow \text{baliR } l \ a \ r = B \ l \ a \ r \\ [\text{ins } x \ l = l; \text{invc } l; \text{invc } r; \text{ins' } x \ l = \text{None}; \text{ins' } x \ r = \text{None}; x < a] \Rightarrow \text{baliL } l \ a \ r = B \ l \ a \ r \\ [\text{ins } x \ l = l; \text{invc } l; \text{invc } r; \text{ins' } x \ l = \text{None}; \text{ins' } x \ r = \text{Some } x2; x < a] \Rightarrow \text{baliL } l \ a \ r = B \ l \ a \ r \end{cases}$ 
2.  $\forall x \ l \ a \ r.$ 
3.  $\forall x \ l \ a \ r \ x2.$ 
4.  $\forall x \ l \ a \ r \ x2.$ 

Output Query Sledgehammer Symbols
34.34 (1004/1023) (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 29/1.269MB 12:41 PM

```

```

Isabelle2017 - RBT.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
RBT.thy (~/ISABELLE_HOME/src/HOL/Data_Structures/)
16 fun baliL :: "'a rbt => 'a => 'a rbt => 'a rbt" where
17 "baliL (R (R t1 a1 t2) a2 t3) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4)" |
18 "baliL (R t1 a1 (R t2 a2 t3)) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4)" |
19 "baliL t1 a t2 = B t1 a t2"
20
21 fun baliR :: "'a rbt => 'a => 'a rbt => 'a rbt" where
22 "baliR t1 a1 (R (R t2 a2 t3) a3 t4) = R (B t1 a1 t2) a2 (B t3 a3 t4)" |
23 "baliR t1 a1 (R t2 a2 (R t3 a3 t4)) = R (B t1 a1 t2) a2 (B t3 a3 t4)" |
24 "baliR t1 a t2 = B t1 a t2"

consts
baliR :: "('a, color) tree => 'a => ('a, color) tree => ('a, color) tree"
Found termination order: "{}"

Output Query Sledgehammer Symbols
21.5 (485/1835) null parsing complete, 0 error(s) (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 24/1.269MB 12:42 PM

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None)
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33 apply (induction x t rule: ins'.induct)
34 apply (auto split: option.split)
35
36 (**)
37 end

proof (prove)
goal (4 subgoals):
1.  $\forall x \ l \ a \ r. \begin{cases} [\text{ins } x \ r = r; \text{invc } l; \text{invc } r; \text{ins' } x \ l = \text{None}; \text{ins' } x \ r = \text{None}; a < x] \Rightarrow \text{baliR } l \ a \ r = B \ l \ a \ r \\ [\text{ins } x \ l = l; \text{invc } l; \text{invc } r; \text{ins' } x \ l = \text{None}; \text{ins' } x \ r = \text{None}; x < a] \Rightarrow \text{baliL } l \ a \ r = B \ l \ a \ r \\ [\text{ins } x \ l = l; \text{invc } l; \text{invc } r; \text{ins' } x \ l = \text{None}; \text{ins' } x \ r = \text{Some } x2; x < a] \Rightarrow \text{baliL } l \ a \ r = B \ l \ a \ r \end{cases}$ 
2.  $\forall x \ l \ a \ r.$ 
3.  $\forall x \ l \ a \ r \ x2.$ 
4.  $\forall x \ l \ a \ r \ x2.$ 

Output Query Sledgehammer Symbols
34.34 (1004/1023) null parsing complete, 0 error(s) (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 24/1.269MB 12:42 PM

```



```

Isabelle2017 - RBT.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
RBT.thy (~$ISABELLE_HOME/src/HOLData_Structures/)
16 fun baliL :: "'a rbt => 'a => 'a rbt" where
17   "baliL (R (R t1 a1 t2) a2 t3) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4) |
18   "baliL (R t1 a1 (R t2 a2 t3)) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4) |
19   "baliL t1 a t2 = B t1 a t2"
20
21 fun baliR :: "'a rbt => 'a => 'a rbt" where
22   "baliR t1 a1 (R (R t2 a2 t3) a3 t4) = R (B t1 a1 t2) a2 (B t3 a3 t4) |
23   "baliR t1 a1 (R t2 a2 (R t3 a3 t4)) = R (B t1 a1 t2) a2 (B t3 a3 t4) |
24   "baliR t1 a t2 = B t1 a t2"

```

consts
baliR :: "('a, color) tree => 'a => ('a, color) tree => ('a, color) tree"
Found termination order: "{}"

21.5 (485/1835) null parsing complete, 0 error(s) (isabelle.isabelle.UTF-8-isabelle)NmroUG 363/1269MB 12:43 PM

```

Isabelle2017 - ex09tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
30 EQ => None"
31
32 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
33   apply (induction x t rule: ins'.induct)
34   apply (auto split: option.split)
35
36 (*<*)
37 end
38 (*>*)

```

proof (prove)
goal (4 subgoals):
1. $\bigwedge x \ l \ a \ r. [ins \ x \ r = r; invc \ l; invc \ r; ins' \ x \ l = None; ins' \ x \ r = None; a < x] \implies baliR \ l \ a \ r = B \ l \ a \ r$
2. $\bigwedge x \ l \ a \ r. [ins \ x \ l = l; invc \ l; invc \ r; ins' \ x \ l = None; ins' \ x \ r = None; x < a] \implies baliL \ l \ a \ r = B \ l \ a \ r$
3. $\bigwedge x \ l \ a \ r \ x2. [ins \ x \ l = l; invc \ l; invc \ r; ins' \ x \ l = None; ins' \ x \ r = Some \ x2; x < a] \implies baliL \ l \ a \ r = B \ l \ a \ r$
4. $\bigwedge x \ l \ a \ r \ x2.$

35.1 (1008/1023) (isabelle.isabelle.UTF-8-isabelle)NmroUG 417/1269MB 12:44 PM

```

Isabelle2017 - ex09tmpl.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
31 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
32   apply (induction x t rule: ins'.induct)
33   apply (auto split: option.split)
34   sledgehammer
35
36 (*<*)
37 end
38 (*>*)

```

Sledgehammering...
Proof found...
"cvc4": Try this: apply (smt RBT_Set.color.simps(2) baliR.elims baliR.simps(4) color.distinct(1) invc.elims(2) tree.inject)

35.15 (1020/1038) (isabelle.isabelle.UTF-8-isabelle)NmroUG 450/1269MB 12:44 PM

```

Isabelle2017 - ex09tmpl.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
31 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
32   apply (induction x t rule: ins'.induct)
33   apply (auto split: option.split)
34   apply (smt RBT_Set.color.simps(2) baliR.elims baliR.simps(4) color.distinct(1) invc.elims(2) tree.inject)
35
36 (*<*)
37 end
38 (*>*)

```

proof (prove)
goal (4 subgoals):
1. $\bigwedge x \ l \ a \ r. [ins \ x \ r = r; invc \ l; invc \ r; ins' \ x \ l = None; ins' \ x \ r = None; a < x] \implies baliR \ l \ a \ r = B \ l \ a \ r$
2. $\bigwedge x \ l \ a \ r. [ins \ x \ l = l; invc \ l; invc \ r; ins' \ x \ l = None; ins' \ x \ r = None; x < a] \implies baliL \ l \ a \ r = B \ l \ a \ r$
3. $\bigwedge x \ l \ a \ r \ x2. [ins \ x \ l = l; invc \ l; invc \ r; ins' \ x \ l = None; ins' \ x \ r = Some \ x2; x < a] \implies baliL \ l \ a \ r = B \ l \ a \ r$
4. $\bigwedge x \ l \ a \ r \ x2.$

35.49 (1054/1131) (isabelle.isabelle.UTF-8-isabelle)NmroUG 500/1269MB 12:45 PM

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
31 lemma "invc t  $\Rightarrow$  case ins' x t of None  $\Rightarrow$  ins x t = t | Some t'  $\Rightarrow$  ins x t = t'"
32
33 apply ((induction x t rule: ins'.induct))
34
35 apply (auto split: option.split)
36
37 (* $\ast$ )
38 end
39 (* $\ast$ )
40
41 proof (prove)
42 goal (3 subgoals):
43 1.  $\wedge x. \text{ invc } () \Rightarrow \text{ case ins' } x () \text{ of None } \Rightarrow \text{ ins } x () = () \mid \text{ Some } xa \Rightarrow \text{ ins } x () = xa$ 
44 2.  $\wedge x \ l \ a \ r. [ [ \text{cmp } x \ a = \text{LT}; \text{ invc } l ] \Rightarrow \text{ case ins' } x \ l \ \text{ of None } \Rightarrow \text{ ins } x \ l = l \mid \text{ Some } xa \Rightarrow \text{ ins } x \ l = xa;$ 
45 [  $\text{cmp } x \ a = \text{GT}; \text{ invc } r ] \Rightarrow \text{ case ins' } x \ r \ \text{ of None } \Rightarrow \text{ ins } x \ r = r \mid \text{ Some } xa \Rightarrow \text{ ins } x \ r = xa;$ 
46  $\text{ invc } (B \ l \ a \ r) ] \Rightarrow \text{ case ins' } x \ (B \ l \ a \ r) \ \text{ of None } \Rightarrow \text{ ins } x \ (B \ l \ a \ r) = B \ l \ a \ r \mid \text{ Some } xa \Rightarrow \text{ ins } x \ (B \ l \ a \ r) = xa$ 
47 3.  $\wedge x \ l \ a \ r.$ 
48
49 Output Query Sledgehammer Symbols
33.42 (970/1023) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 659/1.269MB 12:45 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
34 case (1 x)
35 then show ?case by auto
36 next
37 case (2 x l a r)
38 then show ?case
39 apply (auto split: option.split)
40
41 next
42 case (3 x l a r)
43 then show ?case sorry
44 qed
45
46 proof (prove)
47 goal (4 subgoals):
48 1. [  $\text{ins } x \ r = r; \text{ invc } l; \text{ invc } r; \text{ ins' } x \ l = \text{None}; \text{ ins' } x \ r = \text{None}; a < x ] \Rightarrow \text{ baliR } l \ a \ r = B \ l \ a \ r$ 
49 2. [  $\text{ins } x \ l = l; \text{ invc } l; \text{ invc } r; \text{ ins' } x \ l = \text{None}; \text{ ins' } x \ r = \text{None}; x < a ] \Rightarrow \text{ baliL } l \ a \ r = B \ l \ a \ r$ 
50 3.  $\wedge x2. [ \text{ins } x \ l = l; \text{ invc } l; \text{ invc } r; \text{ ins' } x \ l = \text{None}; \text{ ins' } x \ r = \text{Some } x2; x < a ] \Rightarrow \text{ baliL } l \ a \ r = B \ l \ a \ r$ 
51 4.  $\wedge x2. [ \text{ins } x \ r = r; \text{ invc } l; \text{ invc } r; \text{ ins' } x \ l = \text{Some } x2; \text{ ins' } x \ r = \text{None}; a < x ] \Rightarrow \text{ baliR } l \ a \ r = B \ l \ a \ r$ 
52
53 Output Query Sledgehammer Symbols
38.18 (1049/1192) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 320/1.268MB 12:46 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
35 then show ?case by auto
36 next
37 case (2 x l a r)
38 then show ?case
39 apply (auto split: option.splits)
40 apply (cases "(l,a,r)" rule: baliR.cases; simp)
41 apply (cases "(l,a,r)" rule: baliL.cases; simp)
42 apply (cases "(l,a,r)" rule: baliL.cases; simp)
43 apply (cases "(l,a,r)" rule: baliR.cases; simp)
44 done
45 next
46
47 proof (prove)
48 using this:
49 [  $\text{cmp } x \ a = \text{LT}; \text{ invc } l ] \Rightarrow \text{ case ins' } x \ l \ \text{ of None } \Rightarrow \text{ ins } x \ l = l \mid \text{ Some } a \Rightarrow \text{ ins } x \ l = a$ 
50 [  $\text{cmp } x \ a = \text{GT}; \text{ invc } r ] \Rightarrow \text{ case ins' } x \ r \ \text{ of None } \Rightarrow \text{ ins } x \ r = r \mid \text{ Some } a \Rightarrow \text{ ins } x \ r = a$ 
51  $\text{ invc } (B \ l \ a \ r)$ 
52
53 goal (1 subgoal):
54 1.  $\text{ case ins' } x \ (B \ l \ a \ r) \ \text{ of None } \Rightarrow \text{ ins } x \ (B \ l \ a \ r) = B \ l \ a \ r \mid \text{ Some } b \Rightarrow \text{ ins } x \ (B \ l \ a \ r) = b$ 
55
56 Output Query Sledgehammer Symbols
38.18 (1049/1409) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 797/2.70MB 12:48 PM

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```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
38 show ?case
39 proof (cases "x < a")
40 case True
41 with 2 show ?thesis
42 apply (auto split: option.splits)
43 apply (cases "(l,a,r)" rule: baliL.cases; simp)
44
45 oops
46 apply (cases "(l,a,r)" rule: baliR.cases; simp)
47 apply (cases "(l,a,r)" rule: baliL.cases; simp)
48
49 proof (prove)
50 goal (1 subgoal):
51 1. [  $x < a; \text{ ins' } x \ l = \text{None}; \text{ invc } l; \text{ invc } r; \text{ ins } x \ l = l ] \Rightarrow \text{ baliL } l \ a \ r = B \ l \ a \ r$ 
52
53 Output Query Sledgehammer Symbols
42.39 (1143/1533) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 42/1.270MB 12:51 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
36 then show ?case by auto
37 next
38 case (2 x l a r)
39 then show ?case
40 apply (auto split: option.splits)
41
42 proof (cases "x < a")
43 case True
44 with 2 show ?thesis
45 apply (auto split: option.splits)
46 apply (cases "(l,a,r)" rule: baliL.cases; simp)

proof (prove)
goal (4 subgoals):
1. [ins x r = r; invc l; invc r; ins' x l = None; ins' x r = None; a < x] ==> baliR l a r = B l a r
2. [ins x l = l; invc l; invc r; ins' x l = None; ins' x r = None; x < a] ==> baliL l a r = B l a r
3. [x2. [ins x l = l; invc l; invc r; ins' x l = None; ins' x r = Some x2; x < a] ==> baliL l a r = B l a r
4. [x2. [ins x r = r; invc l; invc r; ins' x l = Some x2; ins' x r = None; a < x] ==> baliR l a r = B l a r

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
36 next
37 case (2 x l a r)
38
39 have aux1: "[invc l; invc r] ==> baliR l a r = B l a r"
40 by (cases "(l,a,r)" rule: baliR.cases; simp)
41
42 have aux2: "[invc l; invc r] ==> baliL l a r = B l a r"
43 by (cases "(l,a,r)" rule: baliL.cases; simp)
44
45 from 2 show ?case
46 by (auto split: option.splits simp: aux1 aux2)

proof (prove)
goal (1 subgoal):
1. [invc l; invc r] ==> baliR l a r = B l a r

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
36 next
37 case (2 x l a r)
38
39 have aux1: "[invc l; invc r] ==> baliR l a r = B l a r"
40 by (cases "(l,a,r)" rule: baliR.cases; simp)
41
42 have aux2: "[invc l; invc r] ==> baliL l a r = B l a r"
43 by (cases "(l,a,r)" rule: baliL.cases; simp)
44
45 from 2 show ?case
46 by (auto split: option.splits simp: aux1 aux2)

proof (prove)
goal (1 subgoal):
1. [invc l; invc r] ==> baliR l a r = B l a r

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
36 next
37 case (2 x l a r)
38
39 have aux1: "[invc l; invc r] ==> baliR l a r = B l a r"
40 by (cases "(l,a,r)" rule: baliR.cases; simp)
41
42 have aux2: "[invc l; invc r] ==> baliL l a r = B l a r"
43 by (cases "(l,a,r)" rule: baliL.cases; simp)
44
45 from 2 show ?case
46 by (auto split: option.splits simp: aux1 aux2)

proof (prove)
goal (1 subgoal):
1. [invc l; invc r] ==> baliR l a r = B l a r

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
27 (case cmp x a of
28   LT => (case ins' x l of None => None | Some l' => Some (R l' a r)) |
29   GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30   EQ => None)
31
32 lemma [simp]: "[invc r] => baliR l a r = B l a r"
33   by (cases "(l,a,r)" rule: baliR.cases; simp)
34
35 lemma [simp]: "[invc l] => baliL l a r = B l a r"
36   by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
39   by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 (**)
42 end

proof (prove)
goal (1 subgoal):
1. invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
EQ => None)
31
32 lemma [simp]: "[invc r] => baliR l a r = B l a r"
33   by (cases "(l,a,r)" rule: baliR.cases; simp)
34
35 lemma [simp]: "[invc l] => baliL l a r = B l a r"
36   by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
39   by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 (**)
42 end
43 (**)
44

theorem invc ?t => case ins' ?x ?t of None => ins ?x ?t = ?t | Some t' => ins ?x ?t = t'

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
29 GT => (case ins' x r of None => None | Some r' => Some (R l a r')) |
30 EQ => None)
31
32 lemma [simp]: "[invc r] => baliR l a r = B l a r"
33   by (cases "(l,a,r)" rule: baliR.cases; simp)
34
35 lemma [simp]: "[invc l] => baliL l a r = B l a r"
36   by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
39   by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 (**)
42 end
43 (**)
44

proof (prove)
goal (1 subgoal):
1. invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
34
35 lemma [simp]: "[invc l] => baliL l a r = B l a r"
36   by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t' ^ ins x t ≠ t'"
39   by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 (**)
42 end
43 (**)
44

theorem invc ?t => case ins' ?x ?t of None => ins ?x ?t = ?t | Some t' => ins ?x ?t = t' ^ ins ?x ?t ≠ ?t
Failed to finish proof:
goal (4 subgoals):
1.  $\forall x l a r.$ 
   [invc l; invc r; ins' x l = None; ins' x r = Some (ins x r); a < x; baliR l a (ins x r) = B l a r;
    ins x r ≠ r]
   => False

```

```

Isabelle2017 - RBT.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
RBT.thy (~$ISABELLE_HOME/src/HOL/Data_Structures/)
16 fun baliL :: "'a rbt => 'a => 'a rbt => 'a rbt" where
17 "baliL (R (R t1 a1 t2) a2 t3) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4) |
18 "baliL (R t1 a1 (R t2 a2 t3)) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4) |
19 "baliL t1 a t2 = B t1 a t2"
20
21 fun baliR :: "'a rbt => 'a => 'a rbt => 'a rbt" where
22 "baliR t1 a1 (R (R t2 a2 t3) a3 t4) = R (B t1 a1 t2) a2 (B t3 a3 t4) |
23 "baliR t1 a1 (R t2 a2 (R t3 a3 t4)) = R (B t1 a1 t2) a2 (B t3 a3 t4) |
24 "baliR t1 a t2 = B t1 a t2"
25
26 fun paint :: "color => 'a rbt => 'a rbt" where
27
consts
baliR :: "('a, color) tree => 'a => ('a, color) tree => ('a, color) tree"
Found termination order: "{}"

```

```

Isabelle2017 - ex09tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmpl.thy (~lehre/FDS/SS18/public/exercises/)
34 lemma [simp]: "[invc l] => baliL l a r = B l a r"
35 by (cases "(l,a,r)" rule: baliL.cases; simp)
36
37 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t' ^ ins x t ≠ t"
38 by (induction x t rule: ins'.induct) (auto split: option.split)
39
40 (*<*)
41 end
42 (*>*)
43
44
proof (prove)
goal (1 subgoal):
1. invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t' ^ ins x t ≠ t

```

```

Isabelle2017 - ex09tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmpl.thy (~lehre/FDS/SS18/public/exercises/)
34 lemma [simp]: "[invc l] => baliL l a r = B l a r"
35 by (cases "(l,a,r)" rule: baliL.cases; simp)
36
37 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t' ^ ins x t ≠ t"
38 by (induction x t rule: ins'.induct) (auto split: option.split)
39
40 (*<*)
41 end
42 (*>*)
43
44
theorem invc ?t => case ins' ?x ?t of None => ins ?x ?t = ?t | Some t' => ins ?x ?t = t' ^ ins ?x ?t ≠ ?t
Failed to finish proof:
goal (4 subgoals):
1. ^x l a r.
[invc l; invc r; ins' x l = None; ins' x r = Some (ins x r); a < x; baliR l a (ins x r) = B l a r;
ins x r ≠ r]
=> False

```

```

Isabelle2017 - ex09tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmpl.thy (~lehre/FDS/SS18/public/exercises/)
36 by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t => case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
39 by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 lemma "invc t => ins x t = t => ins' x t = None"
42
43 (*<*)
44 end
45 (*>*)
46
proof (prove)
goal (1 subgoal):
1. [invc t; ins x t = t] => ins' x t = None

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
36 by (cases "(l,a,r)" rule: balil.cases; simp)
37
38 lemma "invc t  $\implies$  case ins' x t of None  $\implies$  ins x t = t | Some t'  $\implies$  ins x t = t'"
39 by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 lemma "invc t  $\implies$  ins x t = t  $\implies$  ins' x t = None"
42 apply (induction x t rule: ins'.induct)
43 apply (auto split: option.split)
44
45
46 (***)

proof (prove)
goal (6 subgoals):
1.  $\bigwedge x l a r x2.$ 
   [ins x r = r  $\implies$  False; balilR l a (ins x r) = B l a r; invc l; invc r; ins' x l = None;
   ins' x r = Some x2; a < x]
 $\implies$  False
2.  $\bigwedge x l a r x2.$ 

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
36 by (cases "(l,a,r)" rule: balil.cases; simp)
37
38 lemma "invc t  $\implies$  case ins' x t of None  $\implies$  ins x t = t | Some t'  $\implies$  ins x t = t'"
39 by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 lemma "invc t  $\implies$  ins x t = t  $\implies$  ins' x t = None"
42 apply (induction x t rule: ins'.induct)
43 apply (auto split: option.split)
44
45
46 (***)

proof (prove)
goal (6 subgoals):
1.  $\bigwedge x l a r x2.$ 
   [ins x r = r  $\implies$  False; balilR l a (ins x r) = B l a r; invc l; invc r; ins' x l = None;
   ins' x r = Some x2; a < x]
 $\implies$  False
2.  $\bigwedge x l a r x2.$ 

```

```

Isabelle2017 - RBT.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
RBT.thy (~/ISABELLE_HOME/src/HOL/Data_Structures/)
16 fun balil :: "'a rbt  $\implies$  'a  $\implies$  'a rbt  $\implies$  'a rbt" where
17 "balil (R (R t1 a1 t2) a2 t3) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4) |"
18 "balil (R t1 a1 (R t2 a2 t3)) a3 t4 = R (B t1 a1 t2) a2 (B t3 a3 t4) |"
19 "balil t1 a t2 = B t1 a t2"
20
21 fun balilR :: "'a rbt  $\implies$  'a  $\implies$  'a rbt  $\implies$  'a rbt" where
22 "balilR t1 a1 (R (R t2 a2 t3) a3 t4) = R (B t1 a1 t2) a2 (B t3 a3 t4) |"
23 "balilR t1 a1 (R t2 a2 (R t3 a3 t4)) = R (B t1 a1 t2) a2 (B t3 a3 t4) |"
24 "balilR t1 a t2 = B t1 a t2"

consts
balilR :: "('a, color) tree  $\implies$  'a  $\implies$  ('a, color) tree  $\implies$  ('a, color) tree"
Found termination order: "{}"

```

```

Isabelle2017 - ex09tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
37
38 lemma "invc t  $\implies$  case ins' x t of None  $\implies$  ins x t = t | Some t'  $\implies$  ins x t = t'"
39 by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 lemma "size (balilR l x r) = si".
42
43 lemma "invc t  $\implies$  ins' x t = Some t'  $\implies$  size t < size t'"
44 apply (induction x t rule: ins'.induct)
45 apply (auto split: option.splits_if_splits)

Inner syntax error: unexpected end of input
Failed to parse prop

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
44 lemma [simp]: "size (baliL l x r) = size l + size r + 1"
45   apply (cases "(l,x,r)" rule: baliL.cases) by auto
46
47 lemma "invc t  $\implies$  ins' x t = Some t'  $\implies$  size t < size t'"
48   apply (induction x t rule: ins'.induct)
49   apply (auto split: option.splits if_splits)
50
51
52 (* $\ast$ *)

proof (prove)
goal (8 subgoals):
1.  $\bigwedge x l a r x2.$ 
   [x2 = baliR l a x2  $\implies$  size r < size x2; ins' x l = None; invc l; invc r; ins' x r = Some x2; a < x;
   t' = baliR l a x2]
 $\implies$  size r < size x2
2.  $\bigwedge x l a r x2.$ 
   [x2 = baliL x2 a r  $\implies$  size l < size x2; invc l; invc r; ins' x l = Some x2; ins' x r = None; x < a;
   t' = baliL x2 a r]

49.15 (1.521/1571) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 59/1.029MB 1:12 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
44 lemma [simp]: "size (baliL l x r) = size l + size r + 1"
45   apply (cases "(l,x,r)" rule: baliL.cases) by auto
46
47 lemma "invc t  $\implies$  ins' x t = Some t'  $\implies$  size t < size t'"
48   apply (induction x t rule: ins'.induct)
49   apply (auto split: option.splits if_splits)
50
51
52 (* $\ast$ *)

theorem size (baliL ?l ?x ?r) = size ?l + size ?r + 1

46.1 (1.409/1571) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 72/1.029MB 1:12 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
32 lemma [simp]: "[invc r]  $\implies$  baliR l a r = B l a r"
33   by (cases "(l,a,r)" rule: baliR.cases; simp)
34
35 lemma [simp]: "[invc l]  $\implies$  baliL l a r = B l a r"
36   by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t  $\implies$  case ins' x t of None  $\implies$  ins x t = t | Some t'  $\implies$  ins x t = t'"
39   by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 lemma [simp]: "size (baliR l x r) = size l + size r + 1"
42   apply (cases "(l,x,r)" rule: baliR.cases) by auto
43
44 lemma [simp]: "size (baliL l x r) = size l + size r + 1"
45   apply (cases "(l,x,r)" rule: baliL.cases) by auto

42.1 (1.247/1591) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 82/1.011MB 1:13 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
41 lemma [simp]: "size (baliR l x r) = size l + size r + 1"
42   apply (cases "(l,x,r)" rule: baliR.cases) by auto
43
44 lemma [simp]: "size (baliL l x r) = size l + size r + 1"
45   apply (cases "(l,x,r)" rule: baliL.cases) by auto
46
47 lemma "invc t  $\implies$  ins' x t = Some t'  $\implies$  size t < size t'"
48   apply (induction x t arbitrary: t' rule: ins'.induct)
49   apply (auto split: option.splits if_splits)
50   done
51
52 (* $\ast$ *)
53 end
54 (* $\ast$ *)

theorem [invc ?t; ins' ?x ?t = Some ?t']  $\implies$  size ?t < size ?t'

47.1 (1.410/1591) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 51/1.011MB 1:14 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
46
47 lemma "invc t ==> ins' x t = Some t' ==> size t < size t'"
48   apply (induction x t arbitrary: t' rule: ins'.induct)
49   apply (auto split: option.splits if_splits)
50   done
51
52 lemma "invc t ==> ins' x t = Some t' ==> t ≠ t'"
53   apply (induction x t arbitrary: t' rule: ins'.induct)
54   apply (auto split: option.splits if_splits)
55   done
56
proof (prove)
goal (4 subgoals):
1.  $\bigwedge x \ l \ a \ r \ x2. \ [\bigwedge t'. \ x2 = t' \Rightarrow r \neq t'; \text{ins}' \ x \ l = \text{None}; \text{invc } l; \text{invc } r; \text{ins}' \ x \ r = \text{Some } x2; a < x; \ B \ l \ a \ r = \text{baliR } l \ a \ x2] \Rightarrow \text{False}$ 
2.  $\bigwedge x \ l \ a \ r \ x2. \ [\bigwedge t'. \ x2 = t' \Rightarrow l \neq t'; \text{invc } l; \text{invc } r; \text{ins}' \ x \ l = \text{Some } x2; \text{ins}' \ x \ r = \text{None}; x < a; \ B \ l \ a \ r = \text{baliL } x2 \ a \ r]$ 
54.46 (1.721/1.748) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1.41/995MB 1:15 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
48
49 apply (induction x t arbitrary: t' rule: ins'.induct)
50 apply (auto split: option.splits if_splits)
51 done
52
53 lemma "invc t ==> ins' x t = Some t' ==> t ≠ t'"
54   apply (induction x t arbitrary: t' rule: ins'.induct)
55   apply (auto split: option.splits if_splits)
56   done
57
proof (prove)
goal (4 subgoals):
1.  $\bigwedge x \ l \ a \ r \ x2. \ [\bigwedge t'. \ x2 = t' \Rightarrow r \neq t'; \text{ins}' \ x \ l = \text{None}; \text{invc } l; \text{invc } r; \text{ins}' \ x \ r = \text{Some } x2; a < x; \ B \ l \ a \ r = \text{baliR } l \ a \ x2] \Rightarrow \text{False}$ 
2.  $\bigwedge x \ l \ a \ r \ x2. \ [\bigwedge t'. \ x2 = t' \Rightarrow l \neq t'; \text{invc } l; \text{invc } r; \text{ins}' \ x \ l = \text{Some } x2; \text{ins}' \ x \ r = \text{None}; x < a; \ B \ l \ a \ r = \text{baliL } x2 \ a \ r]$ 
52.42 (1.616/1.748) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1.54/995MB 1:16 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
34
35 lemma [simp]: "[invc l] ==> baliL l a r = B l a r"
36   by (cases "(l,a,r)" rule: baliL.cases; simp)
37
38 lemma "invc t ==> case ins' x t of None => ins x t = t | Some t' => ins x t = t'"
39   by (induction x t rule: ins'.induct) (auto split: option.split)
40
41 lemma [simp]: "size (baliR l x r) = size l + size r + 1"
42   apply (cases "(l,x,r)" rule: baliR.cases) by auto
43
proof (prove)
goal (1 subgoal):
1.  $\text{invc } t \Rightarrow \text{case ins}' \ x \ t \text{ of None} \Rightarrow \text{ins } x \ t = t \mid \text{Some } t' \Rightarrow \text{ins } x \ t = t'$ 
38.1 (1.045/1.594) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1.05/995MB 1:16 PM

```

```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex09tmplthy (~/lehre/FDS/SS18/public/exercises/)
49
50 apply (auto split: option.splits if_splits)
51 done
52
53 lemma "invc t ==> case ins' x t of None => ins x t = t | Some t' => ins x t = t' ^ t ≠ t'"
54   using X1[of t x] X2[of t x] by (auto split: option.split)
55
56
57 (**)
58 end
59
proof (prove)
goal (1 subgoal):
1.  $\text{invc } t \Rightarrow \text{case ins}' \ x \ t \text{ of None} \Rightarrow \text{ins } x \ t = t \mid \text{Some } t' \Rightarrow \text{ins } x \ t = t' \wedge t \neq t'$ 
53.84 (1.667/1.747) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 80/964MB 1:17 PM

```



```

Isabelle2017 - ex09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09tmplthy (~/.lehre/FDS/SS18/public/exercises/)
47 lemma X2: "invc t  $\implies$  ins' x t = Some t'  $\implies$  size t < size t'"
48   apply (induction x t arbitrary: t' rule: ins'.induct)
49   apply (auto split: option.splits if_splits)
50   done
51
52
53 lemma "invc t  $\implies$  case ins' x t of None  $\implies$  ins x t = t | Some t'  $\implies$  ins x t = t'  $\wedge$  t  $\neq$  t'"
54   using X1[of t x] X2[of t x] by (auto split: option.split)
55
56
57 (*<*)
58 end
59 (*>*)
60
Proof state Auto update Update Search: 100%
theorem invc ?t  $\implies$  case ins' ?x ?t of None  $\implies$  ins ?x ?t = ?t | Some t'  $\implies$  ins ?x ?t = t'  $\wedge$  ?t  $\neq$  t'
Output Query Sledgehammer Symbols
55.1 (1729/1747) null parsing complete, 0 error(s) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 126/949MB 1:19 PM

```

```

Isabelle2017 - ex09_2tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmplthy (~/.lehre/FDS/SS18/public/exercises/)
7
8
9
10 text <\Exercise{Joining 2-3-Trees}>
11
12 Write a join function for 2-3-trees: The function shall take two
13 2-3-trees <l> and <r> and an element <x>, and return a new 2-3-tree with
14 the inorder-traversal <l x r> .
15
16 Write two functions, one for the height of <l> being greater, the
17 other for the height of <r> being greater.
18
19
20 text <<height r> greater>
21
Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
10.1 (123/1538) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 130/939MB 1:20 PM

```

```

Isabelle2017 - ex09_2tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmplthy (~/.lehre/FDS/SS18/public/exercises/)
12 the inorder-traversal <l x r> .
13
14 Write two functions, one for the height of <l> being greater, the
15 other for the height of <r> being greater.
16
17
18
19
20 text <<height r> greater>
21 fun joinL :: "'a tree23  $\implies$  'a  $\implies$  'a tree23  $\implies$  'a up,"
22 where
23   "joinL l x r = undefined"
24
25 lemma bal_joinL: "[ bal l; bal r; height l  $\leq$  height r ]  $\implies$ 
26   bal (tree_1 (joinL l x r))  $\wedge$  height (tree_1 (joinL l x r)) = height l"
27
Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
22.1 (496/1538) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 54/939MB 1:23 PM

```

```

Isabelle2017 - ex09_2tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmplthy (~/.lehre/FDS/SS18/public/exercises/)
14 Write two functions, one for the height of <l> being greater, the
15 other for the height of <r> being greater.
16
17
18 text <<height l> greater>
19 fun joinR :: "'a tree23  $\implies$  'a  $\implies$  'a tree23  $\implies$  'a up,"
20 where
21   "joinR l x r = undefined"
22
23 lemma bal_joinR: "[ bal l; bal r; height l  $\geq$  height r ]  $\implies$ 
24   bal (tree_1 (joinR l x r))  $\wedge$  height (tree_1 (joinR l x r)) = height l"
25 sorry
26
27 lemma inorder_joinR: "[ bal l; bal r; height l  $\geq$  height r ]  $\implies$  inorder (tree_1 (joinR l x r)) = inorder l @
28   inorder r"
29
Proof state Auto update Update Search: 100%
const
joinR :: "'a tree23  $\implies$  'a  $\implies$  'a tree23  $\implies$  'a up,"
Found termination order: "{}"
Output Query Sledgehammer Symbols
21.27 (526/1540) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 39MB 1:25 PM

```

Isabelle2017 - ex09_2tmpl.thy

```

14 Write two functions, one for the height of <l> being greater, the
15 other for the height of <r> being greater.
16
17
18 text <<height l> greater>
19 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_1"
20 where
21 "joinR l x r = (if height l = height r then Node2 l x r else undefined)"
22
23 lemma bal_joinR: "[ bal l; bal r; height l >= height r ] =>
24 bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"
25 sorry
26
27 lemma inorder_joinR: "[ bal l; bal r; height l >= height r ] => inorder (tree_i (joinR l x r)) = inorder l @
28

```

Type unification failed: Clash of types "_ tree23" and "_ up_1."

Type error in application: incompatible operand type

21.52 (551/1587) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 5/9/12MB 1:26 PM

Isabelle2017 - ex09_2tmpl.thy

```

16
17
18 text <<height l> greater>
19 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_1"
20 where
21 "joinR l x r = (
22   if height l = height r then Up_1 l x r
23   else
24     )"
25
26 lemma bal_joinR: "[ bal l; bal r; height l >= height r ] =>
27 bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"
28

```

Inner syntax errors
Failed to parse prop

23.9 (570/1592) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 5/9/12MB 1:27 PM

Isabelle2017 - ex09_2tmpl.thy (modified)

```

16
17
18 text <<height l> greater>
19 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_1"
20 where
21 "joinR l x r = (
22   if height l = height r then Up_1 l x r
23   else cas
24
25 )"
26
27 lemma bal_joinR: "[ bal l; bal r; height l >= height r ] =>
28 bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"
29

```

Inner syntax errors
Failed to parse prop

23.13 (574/1596) Input/output complete (isabelle.isabelle.UTF-8-isabelle)tmr o UG 5/9/12MB 1:28 PM

Isabelle2017 - ex09_2tmpl.thy

```

17
18 text <<height l> greater>
19 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_1"
20 where
21 "joinR l x r = (
22   if height l = height r then Up_1 l x r
23   else case l of
24     Node2 l1 a l2 => undefined
25     | Node3 l1 a l2 b l3 => undefined
26   )"
27
28 lemma bal_joinR: "[ bal l; bal r; height l >= height r ] =>
29 bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"
30 sorry

```

consts
joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_1"
Found termination order: "{}"

22.24 (542/1669) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 5/9/899MB 1:28 PM

```

Isabelle2017 - ex09_2tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
17
18 text << height l > greater >
19 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
20 where
21 "joinR l x r = (
22   if height l = height r then Up_i l x r
23   else case l of
24     Node2 l1 a l2 => undefined
25     | Node3 l1 a l2 b l3 => undefined
26   )"
27
28 lemma bal_joinR: "[ bal l ; bal r ; height l >= height r ] =>
29   bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"
30 sorry

```

Proof state: Auto update: Update Search: 100%

```

consts
joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
Found termination order: "{}"

```

24.30 (610/1667) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 201/899MB 1:29 PM

```

Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
19
20 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
21 where
22 "joinR l x r = (
23   if height l = height r then Up_i l x r
24   else case l of
25     Node2 l1 a l2 => (case joinR l2 x r of
26       T_i t => undefined
27       | Up_i l2' x' r' => undefined
28       )
29     | Node3 l1 a l2 b l3 => undefined
30   )"
31
32 lemma bal_joinR: "[ bal l ; bal r ; height l >= height r ] =>
33   bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"

```

Proof state: Auto update: Update Search: 100%

```

consts
joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
Found termination order: "(λp. size (fst p)) <+mlex*+ {}"

```

25.26 (650/1750) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 363/887MB 1:31 PM

```

Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
19
20 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
21 where
22 "joinR l x r = (
23   if height l = height r then Up_i l x r
24   else case l of
25     Node2 l1 a l2 => (case joinR l2 x r of
26       T_i t => undefined
27       | Up_i l2' x' r' => undefined
28       )
29     | Node3 l1 a l2 b l3 => undefined
30   )"
31
32 lemma bal_joinR: "[ bal l ; bal r ; height l >= height r ] =>
33   bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"

```

Proof state: Auto update: Update Search: 100%

```

consts
joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
Found termination order: "(λp. size (fst p)) <+mlex*+ {}"

```

25.17 (641/1750) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 363/887MB 1:31 PM

```

Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
19
20 fun joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
21 where
22 "joinR l x r = (
23   if height l = height r then Up_i l x r
24   else case l of
25     Node2 l1 a l2 => (case joinR l2 x r of
26       T_i t => T_i (Node2 l1 a t)
27       | Up_i l2' x' r' => undefined
28       )
29     | Node3 l1 a l2 b l3 => undefined
30   )"
31
32 lemma bal_joinR: "[ bal l ; bal r ; height l >= height r ] =>
33   bal (tree_i (joinR l x r)) ^ height(joinR l x r) = height l"

```

Proof state: Auto update: Update Search: 100%

```

consts
joinR :: "'a tree23 => 'a => 'a tree23 => 'a up_i"
Found termination order: "(λp. size (fst p)) <+mlex*+ {}"

```

26.35 (694/1750) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 46/876MB 1:33 PM

Isabelle2017 - ex09_2tmpl.thy

```

22 if height l = height r then Upi l x r
23 else case l of
24   Node2 l1 a l2 ⇒ (case joinR l2 x r of
25     Ti t ⇒ Ti (Node2 l1 a t)
26     | Upi l2' x' r' ⇒ Ti (Node3 l1 a l2' x' r'))
27   )
28 | Node3 l1 a l2 b l3 ⇒ (case joinR l3 x r of
29   Ti t ⇒ undefined
30   | Upi l2' x' r' ⇒ undefined
31   )
32 )"
33
34 lemma bal_joinR: "[ bal l; bal r; height l ≥ height r ] ⇒
35 bal (treei (joinR l x r)) ∧ height(joinR l x r) = height l"

```

consts
joinR :: "'a tree23 ⇒ 'a ⇒ 'a tree23 ⇒ 'a up_i,"
Found termination order: "(λp. size (fst p)) < *mlex* > {}"

28.33 (752/1855) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 400/365MB 1:34 PM

Isabelle2017 - ex09_2tmpl.thy

```

22 if height l = height r then Upi l x r
23 else case l of
24   Node2 l1 a l2 ⇒ (case joinR l2 x r of
25     Ti t ⇒ Ti (Node2 l1 a t)
26     | Upi l2' x' r' ⇒ Ti (Node3 l1 a l2' x' r'))
27   )
28 | Node3 l1 a l2 b l3 ⇒ (case joinR l3 x r of
29   Ti t ⇒ l1 a l2 b t
30   | Upi l2' x' r' ⇒
31   )"
32 )"
33
34 lemma bal_joinR: "[ bal l; bal r; height l ≥ height r ] ⇒
35 bal (treei (joinR l x r)) ∧ height(joinR l x r) = height l"

```

Inner syntax errors
Failed to parse prop

30.25 (821/1847) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 125/855MB 1:35 PM

Isabelle2017 - ex09_2tmpl.thy (modified)

```

22 if height l = height r then Upi l x r
23 else case l of
24   Node2 l1 a l2 ⇒ (case joinR l2 x r of
25     Ti t ⇒ Ti (Node2 l1 a t)
26     | Upi l2' x' r' ⇒ Ti (Node3 l1 a l2' x' r'))
27   )
28 | Node3 l1 a l2 b l3 ⇒ (case joinR l3 x r of
29   Ti t ⇒ l1 a l2 b t
30   | Upi l2' x' r' ⇒
31   )"
32 )"
33
34 lemma bal_joinR: "[ bal l; bal r; height l ≥ height r ] ⇒
35 bal (treei (joinR l x r)) ∧ height(joinR l x r) = height l"

```

Inner syntax errors
Failed to parse prop

30.26 (822/1848) Input/output complete (isabelle.isabelle.UTF-8-isabelle)tmr o UG 38/855MB 1:36 PM

Isabelle2017 - ex09_2tmpl.thy (modified)

```

22 if height l = height r then Upi l x r
23 else case l of
24   Node2 l1 a l2 ⇒ (case joinR l2 x r of
25     Ti t ⇒ Ti (Node2 l1 a t)
26     | Upi l2' x' r' ⇒ Ti (Node3 l1 a l2' x' r'))
27   )
28 | Node3 l1 a l2 b l3 ⇒ (case joinR l3 x r of
29   Ti t ⇒ l1 a l2 b t
30   | Upi l2' x' r' ⇒ Upi () b ())
31   )"
32 )"
33
34 lemma bal_joinR: "[ bal l; bal r; height l ≥ height r ] ⇒
35 bal (treei (joinR l x r)) ∧ height(joinR l x r) = height l"

```

Type unification failed: Clash of types "_ ⇒ _" and "_ tree23"
Type error in application: operator not of function type

30.38 (834/1860) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 4/855MB 1:36 PM

```
Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
27
28 a l2 b l3 => (case joinR l3 x r of
29 T1 (Node3 l1 a l2 b t)
30 x' r' => Up1 (Node2 l1 a l2) b (Node2 l2' x' r'))
31
32
33
34 inR: "[ bal l; bal r; height l > height r ] => inorder (tree_i (joinR l x r)) = inorder l @ x # inorder r"
35
36
37
38
39 : "[ bal l; bal r; height l > height r ] =>
40 inR l x r) ^ height (joinR l x r) = height l"
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proof (prove)
goal (1 subgoal):
1. [bal l; bal r; height r <= height l] => inorder (tree_i (joinR l x r)) = inorder l @ x # inorder r
end

```

```
Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
27
28 )
29 | Node3 l1 a l2 b l3 => (case joinR l3 x r of
30 T1 t => T1 (Node3 l1 a l2 b t)
31 | Up1 l2' x' r' => Up1 (Node2 l1 a l2) b (Node2 l2' x' r'))
32 )
33 )"
34 lemma inorder_joinR: "[ bal l; bal r; height l > height r ] => inorder (tree_i (joinR l x r)) = inorder l @:
35 sorry
36
37
38
39 lemma bal_joinR: "[ bal l; bal r; height l > height r ] =>
40 bal (tree_i (joinR l x r)) ^ height (joinR l x r) = height l"
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proof (prove)
goal (1 subgoal):
1. [bal l; bal r; height r <= height l] => inorder (tree_i (joinR l x r)) = inorder l @ x # inorder r
end

```

```
Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
33
34 declare joinR.simps[simp del]
35
36 lemma inorder_joinR: "[ bal l; bal r; height l > height r ] => inorder (tree_i (joinR l x r)) = inorder l @:
37 apply (induction l x r rule: joinR.induct)
38 apply (auto split!: tree23.split)
39
40
41
42
43 lemma bal_joinR: "[ bal l; bal r; height l > height r ] =>
44 bal (tree_i (joinR l x r)) ^ height (joinR l x r) = height l"
45 sorry
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100

proof (prove)
goal (1 subgoal):
1. [bal l; bal r; height r <= height l] => inorder (tree_i (joinR l x r)) = inorder l @ x # inorder r
end

```

```
Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
32 )"
33 declare joinR.simps[simp del]
34
35 lemma inorder_joinR: "[ bal l; bal r; height l > height r ] => inorder (tree_i (joinR l x r)) = inorder l @:
36 apply (induction l x r rule: joinR.induct)
37 apply (auto split!: tree23.split)
38 apply (subst joinR.simps)
39
40
41
42
43 lemma bal_joinR: "[ bal l; bal r; height l > height r ] =>
44 bal (tree_i (joinR l x r)) ^ height (joinR l x r) = height l"
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else case l of
<l1, a, l2> =>
case joinR l2 x r of T1 t => T1 <l1, a, t>
| Up1 l2' x' r' => T1 <l1, a, l2', x', r'>
| <l1, a, l2, b, l3> =>
case joinR l3 x r of T1 t => T1 <l1, a, l2, b, t>
end

```

```

Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
37 Lemma inorder_joinR: "[ bal l; bal r; height l > height r ] => inorder (tree_i (joinR l x r)) = inorder l @
38 apply (induction l x r rule: joinR.induct)
39 apply (auto split!: tree23.split)
40 apply (subst joinR.simps)
41 apply (auto split!: tree23.split up_i.split)
42
43
44
[
  \x21a x22a x23a.
  [Suc (max (height x21a) (height x23a)) ≠ height r; x21 = x21a ∧ x22 = x22a ∧ x23 = x23a;
  bal x23a; height r ≤ height x23a]
  => inorder (tree_i (joinR x23a x r)) = inorder x23a @ x # inorder r;
  \x31 x33 x35.
  [Suc (max (height x31) (max (height x33) (height x35))) ≠ height r; False; bal x35;
  height r ≤ height x35]
  => inorder (tree_i (joinR x35 x r)) = inorder x35 @ x # inorder r;
  bal r; height r ≤ Suc (height x23); Suc (height x23) ≠ height r; bal x21; bal x23;
  height x21 = height x23; joinR x23 x r = T_i x1]
]
Output Query Sledgehammer Symbols
41.47 (1197/2083) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 56/771MB 1:45 PM

```

```

Isabelle2017 - ex09_2tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
36 Lemma inorder_joinR: "[ bal l; bal r; height l > height r ] => inorder (tree_i (joinR l x r)) = inorder l @
37 apply (induction l x r rule: joinR.induct)
38 apply (subst joinR.simps)
39 apply (fastforce split!: tree23.split up_i.split)
40 apply fastforce
41 apply fastforce
42 apply fastforce
43 apply fastforce
44
45
proof (prove)
goal:
No subgoals!
Output Query Sledgehammer Symbols
40.19 (1193/2129) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 52/756MB 1:47 PM

```

```

Isabelle2017 - ex09_2tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
38 apply (induction l x r rule: joinR.induct)
39 apply (subst joinR.simps)
40 apply (fastforce split!: tree23.split up_i.split)
41 apply fastforce
42 apply fastforce
43 apply fastforce
44 apply fastforce
45 done
46
proof (prove)
goal:
No subgoals!
Output Query Sledgehammer Symbols
41.1 (1167/2129) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 55/756MB 1:47 PM

```

```

Isabelle2017 - ex09_2tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
28 | Node3 l1 a l2 b l3 => (case joinR l3 x r of
29 | T_i t => T_i (Node3 l1 a l2 b t)
30 | Up_i l2' x' r' => Up_i (Node2 l1 a l2) b (Node2 l2' x' r'))
31 )
32 )"
33
34 declare joinR.simps[simp del]
35
proof (prove)
goal:
No subgoals!
Output Query Sledgehammer Symbols
31.1 (875/2057) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 55/756MB 1:47 PM

```

```

Isabelle2017 - ex09_2tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex09_2tmplthy (~/lehre/FDS/SS18/public/exercises/)
38 apply (induction l x r rule: joinR.induct)
39 apply (subst joinR.simps)
40 apply (fastforce split!: tree23.split up;.split)
41 done
42
43 lemma bal_joinR: "[ bal l; bal r; height l ≥ height r ] ⇒
44 bal (treei (joinR l x r)) ∧ height(joinR l x r) = height l"
45 sorry
46
47
48
49 text <<height r> greater>
50 fun joinL :: "'a tree23 ⇒ 'a ⇒ 'a tree23 ⇒ 'a up;"
51 where
52 "joinL l x r = undefined"

```

theorem
inorder_joinR:
[bal ?l; bal ?r; height ?r ≤ height ?l]
⇒ inorder (tree_i (joinR ?l ?x ?r)) = inorder ?l @ ?x # inorder ?r

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
3 (*>*)
4
5 text <<NumHomework{Balanced Tree to RBT}{15.-6.-2018}>
6
7 A tree is balanced, if its minimum height and its height differ by at most 1.
8
9
10
11
12 fun min_height :: "('a,'b) tree ⇒ nat" where
13 "min_height Leaf = 0" |
14 "min_height (Node _ l _ r) = min (min_height l) (min_height r) + 1"
15
16 definition "balanced t ≡ height t - min_height t ≤ 1"
17
18 text <<The following function paints a balanced tree to form a valid red-black tree
19 with the same structure. The task of this homework is to prove this!>

```

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
21
22 fun mk_rbt :: "('a,unit) tree ⇒ 'a rbt" where
23 "mk_rbt Leaf = Leaf"
24 | "mk_rbt (Node _ l a r) = (let
25   l'=mk_rbt l;
26   r'=mk_rbt r
27 in
28   if min_height l > min_height r then
29     B (paint Red l') a r'
30   else if min_height l < min_height r then
31     B l' a (paint Red r')
32   else
33     B l' a r'
34 )"

```

consts
mk_rbt :: "('a, unit) tree ⇒ ('a, color) tree"
Found termination order: "size < *mlex* > {}"

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
22 fun mk_rbt :: "('a,unit) tree ⇒ 'a rbt" where
23 "mk_rbt Leaf = Leaf"
24 | "mk_rbt (Node _ l a r) = (let
25   l'=mk_rbt l;
26   r'=mk_rbt r
27 in
28   if min_height l > min_height r then
29     B (paint Red l') a r'
30   else if min_height l < min_height r then
31     B l' a (paint Red r')
32   else
33     B l' a r'
34 )"

```

consts
mk_rbt :: "('a, unit) tree ⇒ ('a, color) tree"
Found termination order: "size < *mlex* > {}"

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
23 "mk_rbt Leaf = Leaf"
24 | "mk_rbt (Node _ l a r) = (let
25   l'=mk_rbt l;
26   r'=mk_rbt r
27 in
28   if min_height l > min_height r then
29     B (paint Red l') a r'
30   else if min_height l < min_height r then
31     B l' a (paint Red r')
32   else
33     B l' a r'
34 )"
37 text <
  \subsection{Bonus}
  >
const
mk_rbt :: "('a, unit) tree => ('a, color) tree"
Found termination order: "size <*mlex*> {}"
Output Query Sledgehammer Symbols
29.16 (762/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 72/743MB 1:50 PM

```

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
51 sorry
52
53 text <
54   \subsection{The Easy Parts}
55
56   Show that <mk_rbt> does not change the inorder-traversal
57 >
58 lemma mk_rbt_inorder: "inorder (mk_rbt t) = inorder t"
59 sorry
60
61 text <Show that the color of the root node is always black>
62 lemma mk_rbt_color: "color (mk_rbt t) = Black"
63 sorry
64
65 text <
  \subsection{Medium Complex Parts}
  >
proof (prove)
goal (1 subgoal):
1. inorder (mk_rbt t) = inorder t
Output Query Sledgehammer Symbols
60.1 (1493/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 313/736MB 1:52 PM

```

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
21
22 fun mk_rbt :: "('a,unit) tree => 'a rbt" where
23 "mk_rbt Leaf = Leaf"
24 | "mk_rbt (Node _ l a r) = (let
25   l'=mk_rbt l;
26   r'=mk_rbt r
27 in
28   if min_height l > min_height r then
29     B (paint Red l') a r'
30   else if min_height l < min_height r then
31     B l' a (paint Red r')
32   else
33     B l' a r'
34 )"
37
const
mk_rbt :: "('a, unit) tree => ('a, color) tree"
Found termination order: "size <*mlex*> {}"
Output Query Sledgehammer Symbols
28.1 (707/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 35/724MB 1:54 PM

```

```

Isabelle2017 - hw09tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
hw09tmplthy (~/lehre/FDS/SS18/public/exercises/)
82 sorry
83
84 text <
85   \subsection{The Hard Part (3 Bonus Points)}
86
87   For {\bf three bonus points}, show that the returned tree satisfies the color invariant.
88
89   Warning: This requires careful case splitting, via a clever combination of
90   automation and manual proof (Isar, aux-lemmas), in order to deal with the
91   multiple cases without a combinatorial explosion of the proofs.
92 >
93
94 lemma mk_rbt_inv: "balanced t ==> invc (mk_rbt t)"
95 sorry
96
Output Query Sledgehammer Symbols
90.1 (2373/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 3/724MB 1:54 PM

```



```

Isabelle2017 - hw09tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
hw09tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
82 sorry
83
84 text <
85 \subsection*{The Hard Part (3 Bonus Points)}
86
87 For {\bf three bonus points}, show that the returned tree satisfies the color invariant.
88
89 Warning: This requires careful case splitting, via a clever combination of
90 automation and manual proof (Isar, aux-lemmas), in order to deal with the
91 multiple cases without a combinatorial explosion of the proofs.
92
93
94 Lemma mk_rbt_invc: "balanced t  $\implies$  invc (mk_rbt t)"
95 sorry
96
[ ] Proof state [x] Auto update Update Search: [ ] 100%
[ ] Output Query Sledgehammer Symbols
88.1 (2295/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 235/724MB 1:55 PM

```

```

Isabelle2017 - hw09tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
hw09tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
113 >
114
115 fun mk_rbt' :: "('a,unit) tree  $\implies$  'a rbt  $\times$  nat" -- <Returns the RBT and the min-height of the argument>
116 where
117   "mk_rbt' _ = undefined"
118
119 lemma mk_rbt'_refine_aux: "mk_rbt' t = (mk_rbt t, min_height t)"
120 sorry
121
122
123 lemma mk_rbt'_refine: "fst (mk_rbt' t) = mk_rbt t"
124 sorry
125
126
127
[ ] Proof state [x] Auto update Update Search: [ ] 100%
[ ] Output Query Sledgehammer Symbols
123.1 (3282/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 102/714MB 1:57 PM

```