

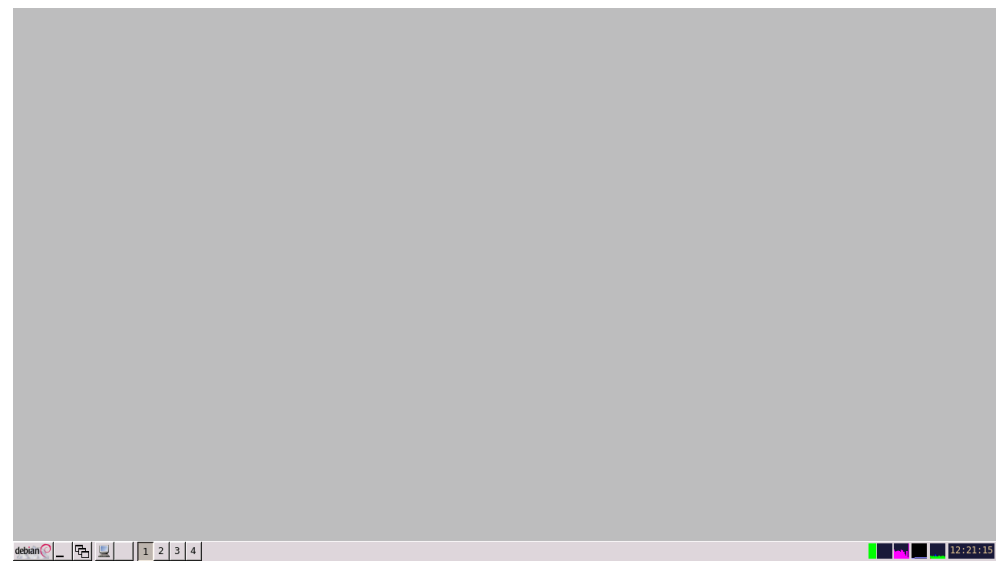
Script generated by TTT

Title: Lammich: FDS Tutorial (01.06.2018)

Date: Fri Jun 01 12:21:15 CEST 2018

Duration: 82:50 min

Pages: 84



```
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$ ls
ex01.pdf  ex07.pdf      tmp103.thy-  tut01.thy-  tut04.thy-  tut07.thy-
ex02.pdf  ex08.pdf      tmp104.thy-  tut01.thy-  tut05.thy-
ex03.pdf  ex08tmpl.thy  tmp104.thy-  tut02.thy-  tut05.thy-
ex04.pdf  hw08bonustpl.thy  tmp105.thy-  tut03.thy-  tut06.thy-
ex05.pdf  hw08tmpl.thy  tmp106.thy-  tut03.thy-  tut06.thy-
ex06.pdf  tmp103.thy    tmp107.thy-  tut04.thy-  tut07.thy-
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$ evince ex08.pdf
```

Technische Universität München
Institut für Informatik
Prof. Tobias Nipkow, Ph.D.
Dr. Peter Lammich

SS 2018
1. 6. 2018

Functional Data Structures

Exercise Sheet 8

Exercise 8.1 Joining 2-3-Trees

Implement and prove correct a function to combine two 2-3-trees of equal height, such that the inorder traversal of the resulting tree is the concatenation of the inorder traversal of the arguments, and the height of the result is either the height of the arguments, or has increased by one. Use `h up` to return the result, similar to `Tree23Set.set`:

```
fun join : "a tree23 => 'a tree23 => 'a up"
```

lemma join_inorder:
Exms 11 11 : "a tree23"
assumes "height l1 = height l2"
assumes "hd l1 = hd l2"
shows "inorder (tree (join l1 l2)) = (inorder l1 @ inorder l2)"

lemma join_hgt:
Exms 11 12 : "a tree23"
assumes "height l1 = height l2"
assumes "hd l1 = hd l2"
shows "hd (tree (join l1 l2)) < height l2"

Hints:

- Try to use automatic case splitting (`auto split: ...`) instead of explicit case splitting via `four` (There will be dozens of cases).

```
lammich@lapnikow10: ~/opt/ttt
lammich@lapnikow10: ~/opt/ttt$ cd ~/lehre/FDS/SS18/public/exercises/
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$ ls
```

Terminal window showing file listing and execution:

```

lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises
File Edit View Search Terminal Help
lammich@lapnikow10: ~/opt/tts$ cd ~/lehre/FDS/SS18/public/exercises/
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$ ls
ex01.pdf ex07.pdf      tmp103.thy-  tut01.thy-  tut04.thy-  tut07.thy-
ex02.pdf ex08.pdf      tmp104.thy-  tut01.thy-  tut05.thy-
ex03.pdf ex08tmp1.thy  tmp104.thy-  tut02.thy-  tut05.thy-
ex04.pdf hw08bonustmp1.thy tmp105.thy-  tut03.thy-  tut06.thy-
ex05.pdf hw08tmp1.thy  tmp106.thy-  tut03.thy-  tut06.thy-
ex06.pdf tmp102.thy  tmp107.thy-  tut04.thy-  tut07.thy-
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$ evince ex08.pdf
[1]+  Stopped                  evince ex08.pdf
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$
[1]-  evince ex08.pdf &
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$
hy

```

PDF viewer showing "Functional Data Structures Exercise Sheet 8" with a logo and text:

Deutsche Universität München SS 2018
 Institut für Informatik 1. 6. 2018
 Prof. Tibian Nipkow, Ph.D.
 Dr. Peter Lammich

Exercise 8.1 Joining 2-3-Trees

Implement and prove correct a function to combine two 2-3-trees of equal height, such that the inorder traversal of the resulting tree is the concatenation of the inorder traversal of arguments, and the height of the result is either the height of the arguments, or increased by one. Use `'a upi'` to return the result, similar to `Tree23_Set.ins`:

```

join :: 'a tree23 => 'a tree23 => 'a upi

```

Terminal window showing the execution of the evince command:

```

[1]+  Stopped                  evince ex08.pdf
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$
[1]-  evince ex08.pdf &
lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises$
hy

```

Isabelle2017 - ex08tmp1.thy editor window showing the initial code:

```

1 (**)
2 theory ex08tmp1
3 imports
4   Complex_Main
5   "HOL-Data_Structures.Tree23_Set"
6 begin
7 (***)
8
9 text {*\ExerciseSheet{8}{1.-6.-2018} *}
10
11 text <\Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the
16 arguments, and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{typ "'a upi'} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 => 'a tree23 => 'a upi"

```

Isabelle2017 - ex08tmp1.thy editor window showing the code after the first lemma:

```

11 text <\Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{typ "'a upi'} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 => 'a tree23 => 'a upi"
22 where
23   "join Leaf Leaf = T1 Leaf"
24
25
26 lemma join_inorder:
27   fixes t1 t2 :: "'a tree23"
28   assumes "height t1 = height t2"
29   assumes "bal t1" "bal t2"
30   shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
31 oops

```

Isabelle2017 - ex08tmp1.thy editor window showing the code after the second lemma:

```

20
21 fun join :: "'a tree23 => 'a tree23 => 'a upi"
22 where
23   "join Leaf Leaf = T1 Leaf"
24
25
26 lemma join_inorder:
27   fixes t1 t2 :: "'a tree23"
28   assumes "height t1 = height t2"
29   assumes "bal t1" "bal t2"
30   shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
31 oops
32
33 lemma join_bal:
34   fixes t1 t2 :: "'a tree23"
35   assumes "height t1 = height t2"
36   assumes "bal t1" "bal t2"
37   shows "bal (treei (join t1 t2)) ^ height (join t1 t2) = height t2"
38 oops
39
40 text <\Hints:

```

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24
25
26 lemma join_inorder:
27   fixes t1 t2 :: "'a tree23"
28   assumes "height t1 = height t2"
29   assumes "bal t1" "bal t2"
30   shows "inorder (tree_i (join t1 t2)) = (inorder t1 @ inorder t2)"
31   oops
32
33 lemma join_bal:
34   fixes t1 t2 :: "'a tree23"
35   assumes "height t1 = height t2"
36   assumes "bal t1" "bal t2"
37   shows "bal (tree_i (join t1 t2)) ^ height (join t1 t2) = height t2"
38   oops
39
40 text <Hints:

```

23.21 (638/1844) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG : 70/1192MB 12:27 PM
 debian | 1 2 3 4 | iamlich@lapnikow10: ~/lehre/FDS/SS1... | ex08.pdf | Isabelle2017 - ex08tmpl.thy | 12:27:37

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24
25
26 lemma join_inorder:
27   fixes t1 t2 :: "'a tree23"
28   assumes "height t1 = height t2"
29   assumes "bal t1" "bal t2"
30   shows "inorder (tree_i (join t1 t2)) = (inorder t1 @ inorder t2)"
31   oops
32
33 lemma join_bal:
34   fixes t1 t2 :: "'a tree23"
35   assumes "height t1 = height t2"
36   assumes "bal t1" "bal t2"
37   shows "bal (tree_i (join t1 t2)) ^ height (join t1 t2) = height t2"
38   oops
39
40 text <Hints:

```

37.62 (1005/1844) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG : 82/1192MB 12:27 PM
 debian | 1 2 3 4 | iamlich@lapnikow10: ~/lehre/FDS/SS1... | ex08.pdf | Isabelle2017 - ex08tmpl.thy | 12:27:55

```

Isabelle2017 - Tree23_Set.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Tree23_Set.thy ($ISABELLE_HOME/src/HOL/Data_Structures/)
18
19   GT => isin r x) |
20   "isin (Node3 l a m b r) x =
21   (case cmp x a of
22     LT => isin l x |
23     EQ => True |
24     GT =>
25       (case cmp x b of
26         LT => isin m x |
27         EQ => True |
28         GT => isin r x))"
29
30 datatype 'a Up_i = T_1 "'a tree23" | Up_i "'a tree23" 'a "'a tree23"
31
32 fun tree_i :: "'a up_i => 'a tree23" where
33   "tree_i (T_1 t) = t" |
34   "tree_i (Up_i l a r) = Node2 l a r"
35
36 fun ins :: "'a::linorder => 'a tree23 => 'a up_i" where
37   "ins x Leaf = Up_i Leaf x Leaf" |
38   "ins x (Node2 l a r) =
39     (case cmp x a of

```

29.13 (524/12937) null parsing complete, 0 error(s) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG : 93/1192MB 12:28 PM
 debian | 1 2 3 4 | iamlich@lapnikow10: ~/lehre/FDS/SS1... | ex08.pdf | Isabelle2017 - Tree23_Set.thy | 12:28:45

```

Isabelle2017 - Tree23_Set.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Tree23_Set.thy ($ISABELLE_HOME/src/HOL/Data_Structures/)
18
19   GT => isin r x) |
20   "isin (Node3 l a m b r) x =
21   (case cmp x a of
22     LT => isin l x |
23     EQ => True |
24     GT =>
25       (case cmp x b of
26         LT => isin m x |
27         EQ => True |
28         GT => isin r x))"
29
30 datatype 'a up_i = T_1 "'a tree23" | Up_i "'a tree23" 'a "'a tree23"
31
32 fun tree_i :: "'a up_i => 'a tree23" where
33   "tree_i (T_1 t) = t" |
34   "tree_i (Up_i l a r) = Node2 l a r"
35
36 fun ins :: "'a::linorder => 'a tree23 => 'a up_i" where
37   "ins x Leaf = Up_i Leaf x Leaf" |
38   "ins x (Node2 l a r) =
39     (case cmp x a of

```

33.36 (682/12937) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG : 6/1192MB 12:29 PM
 debian | 1 2 3 4 | iamlich@lapnikow10: ~/lehre/FDS/SS1... | ex08.pdf | Isabelle2017 - Tree23_Set.thy | 12:29:00

```

Isabelle2017 - Tree23_Set.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Tree23_Set.thy (|SISABELLE_HOME/src/HOL/Data_Structures/)
198 by(simp add: delete_def inorder_del)
199
200
201 subsection <Balancedness>
202
203
204 subsection "Proofs for insert"
205
206 text{* First a standard proof that @{const ins} preserves @{const bal}. *}
207
208 instantiation up_i :: (type)height
209 begin
210
211 fun height_up_i :: "'a up_i ⇒ nat" where
212   "height (T_i t) = height t" |
213   "height (Up_i l a r) = height l"
214
215 instance ..
216
217 end
218
Output Query Sledgehammer Symbols
213.32 (721/12937) (isabelle.isabelle.UTF-8-Isabelle)Nm r o UG 702/11 92MB 12:29 PM

```

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
23 "join Leaf Leaf = T_i Leaf"
24
25
26 lemma join_inorder:
27   fixes t1 t2 :: "'a tree23"
28   assumes "height t1 = height t2"
29   assumes "bal t1" "bal t2"
30   shows "inorder (tree_i (join t1 t2)) = (inorder t1 @ inorder t2)"
31   oops
32
33 lemma join_bal:
34   fixes t1 t2 :: "'a tree23"
35   assumes "height t1 = height t2"
36   assumes "bal t1" "bal t2"
37   shows "bal (tree_i (join t1 t2)) ∧ height (join t1 t2) = height t2"
38   oops
39
40 text <Hints:
41   * Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar
42     (There will be dozens of cases).
43   * To find bugs in your join function, or isolate the case where your automatic proof does not (yet) work,
Output Query Sledgehammer Symbols
36.28 (943/1844) (isabelle.isabelle.UTF-8-Isabelle)Nm r o UG 809/11 2MB 12:30 PM

```

```

Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
10 text <\Exercise{Joining 2-3-Trees}>
11
12 Implement and prove correct a function to combine two
13 2-3-trees of equal height, such that the inorder traversal of the
14 resulting tree is the concatenation of the inorder traversal of the arguments,
15 and the height of the result is either the height of the arguments, or
16 has increased by one. Use @{typ "'a up_i"} to return the result, similar to
17 @{const [source] Tree23_Set.ins}:
18
19 >
20
21 fun join :: "'a tree23 ⇒ 'a tree23 ⇒ 'a up_i"
22 where
23   "join Leaf Leaf = T_i Leaf"
24 |
25 lemma join_inorder:
26   fixes t1 t2 :: "'a tree23"
27   assumes "height t1 = height t2"
28   assumes "bal t1" "bal t2"
29   shows "inorder (tree_i (join t1 t2)) = (inorder t1 @ inorder t2)"
30   oops
Output Query Sledgehammer Symbols
24.3 (650/1845) (isabelle.isabelle.UTF-8-Isabelle)Nm r o UG 26/11 90MB 12:37 PM

```

```

Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <\Exercise{Joining 2-3-Trees}>
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{typ "'a up_i"} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 ⇒ 'a tree23 ⇒ 'a up_i"
22 where
23   "join Leaf Leaf = T_i Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = ("
25 ..
26
27 )"
28
29
30
31
Output Query Sledgehammer Symbols
25.5 (696/1902) Input/output complete (isabelle.isabelle.UTF-8-Isabelle)Nm r o UG 61/1 265MB 12:38 PM

```

```
Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <\Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{\typ "'a up_i"} to return the result, similar to
18 @{\const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   ...
26
27
28 )"
29
30
31
Output Query Sledgehammer Symbols
25.5 (696/1902) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 66/1265MB 12:39 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy (modified) 12:39:00
```

```
Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <\Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{\typ "'a up_i"} to return the result, similar to
18 @{\const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   joi
26
27
28 )"
29
30
31
Output Query Sledgehammer Symbols
25.8 (699/1905) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 75/1265MB 12:40 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy (modified) 12:40:50
```

```
Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <\Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{\typ "'a up_i"} to return the result, similar to
18 @{\const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T_1 t23 => undefined
27   | Up_i t2' x t3' => undefined
28
29 )"
30
31
Output Query Sledgehammer Symbols
26.16 (732/1981) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 69/1265MB 12:41 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:41:46
```

```
Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <\Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{\typ "'a up_i"} to return the result, similar to
18 @{\const [source] Tree23_Set.ins}:
19 >
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T_1 t23 => undefined
27   | Up_i t2' x t3' => undefined
28
29 )"
30
31
Output Query Sledgehammer Symbols
26.17 (733/1981) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 69/1265MB 12:43 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:43:00
```

```
Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
11 text \Exercise{Joining 2-3-Trees}
12
13 Implement and prove correct a function to combine two
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{typ "'a up_i"} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T_1 t23 => undefined
27   | Up_i t2' x t3' => undefined
28   )"
29
30
31
Output Query Sledgehammer Symbols
26.26 (742/1981) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 719/1 65MB 12:43 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 12:43:39
```

```
Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{typ "'a up_i"} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T_1 t23 => T_1 (Node3 t1 a t23 b t4)
27   | Up_i t2' x t3' => Up_i (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29 | "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30
31
32
33
34
Output Query Sledgehammer Symbols
31.2 (879/2079) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 729/1 93MB 12:46 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 12:46:00
```

```
Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
14 2-3-trees of equal height, such that the inorder traversal of the
15 resulting tree is the concatenation of the inorder traversal of the arguments,
16 and the height of the result is either the height of the arguments, or
17 has increased by one. Use @{typ "'a up_i"} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T_1 t23 => T_1 (Node3 t1 a t23 b t4)
27   | Up_i t2' x t3' => Up_i (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29 | "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30
31
32
33
34
Output Query Sledgehammer Symbols
27.3 (761/2079) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 762/2 93MB 12:50 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 12:50:29
```

```
Isabelle2017 - ex08tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
17 has increased by one. Use @{typ "'a up_i"} to return the result, similar to
18 @{const [source] Tree23_Set.ins}:
19
20
21 fun join :: "'a tree23 => 'a tree23 => 'a up_i"
22 where
23   "join Leaf Leaf = T_1 Leaf"
24 | "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T_1 t23 => T_1 (Node3 t1 a t23 b t4)
27   | Up_i t2' x t3' => Up_i (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29 | "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30   case (join t2 t3) of
31     T_1 t23 => Up_i (
32   | Up_i t2' x t3' => Up_i
33   )"
34
35
36
37
Output Query Sledgehammer Symbols
31.24 (925/2155) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 88/1293MB 12:51 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy (modified) 12:51:10
```

```
Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23 "join Leaf Leaf = T1 Leaf"
24 "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node2 t3' b t4)
28 )"
29 "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30   case (join t2 t3) of
31     T1 t23 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
32   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33 )"
34 "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35   case (join t2 t3) of
36     T1 t23 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
37   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
38 )"
39
40
41
42
Output Query Sledgehammer Symbols
34.42 (1.071/2435) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 38.3/1.309MB 12:53 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:53:09
```

```
Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23 "join Leaf Leaf = T1 Leaf"
24 "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node2 t3' b t4)
28 )"
29 "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30   case (join t2 t3) of
31     T1 t23 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
32   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33 )"
34 "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35   case (join t2 t3) of
36     T1 t23 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
37   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
38 )"
39
40
41
42
Output Query Sledgehammer Symbols
34.30 (1.059/2430) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 38.3/1.309MB 12:53 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:53:36
```

```
Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23 "join Leaf Leaf = T1 Leaf"
24 "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node2 t3' b t4)
28 )"
29 "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30   case (join t2 t3) of
31     T1 t23 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
32   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33 )"
34 "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35   case (join t3 t4) of
36     T1 t34 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
37   | Up1 t3' x t4' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
38 )"
39
40
41
42
Output Query Sledgehammer Symbols
34.39 (1.068/2430) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 38.3/1.309MB 12:54 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:54:13
```

```
Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23 "join Leaf Leaf = T1 Leaf"
24 "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25   case (join t2 t3) of
26     T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node2 t3' b t4)
28 )"
29 "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30   case (join t2 t3) of
31     T1 t23 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
32   | Up1 t2' x t3' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33 )"
34 "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35   case (join t3 t4) of
36     T1 t34 ⇒ Up1 (Node2 t1 a t23) b (Node2 t4 c t5)
37   | Up1 t3' x t4' ⇒ Up1 (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
38 )"
39
40
41
42
Output Query Sledgehammer Symbols
36.14 (1.143/2430) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 38.3/1.309MB 12:54 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:54:27
```

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23   "join Leaf Leaf = T1 Leaf"
24   "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25     case (join t2 t3) of
26       T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29   "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30     case (join t2 t3) of
31       T1 t23 ⇒ Upi (Node2 t1 a t23) b (Node2 t4 c t5)
32     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33   )"
34   "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35     case (join t3 t4) of
36       T1 t34 ⇒ Upi (Node2 t1 a t2) b (Node2 t34 c t5)
37     | Upi t3' x t4' ⇒ Upi (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
38   )"
39
40
41
42
Output Query Sledgehammer Symbols
37.30 (1.189/2430) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 98/1310MB 12:55 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:55:07

```

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23   "join Leaf Leaf = T1 Leaf"
24   "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25     case (join t2 t3) of
26       T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29   "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30     case (join t2 t3) of
31       T1 t23 ⇒ Upi (Node2 t1 a t23) b (Node2 t4 c t5)
32     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33   )"
34   "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35     case (join t3 t4) of
36       T1 t34 ⇒ Upi (Node2 t1 a t2) b (Node2 t34 c t5)
37     | Upi t3' x t4' ⇒ Upi (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
38   )"
39
40
41
42
Output Query Sledgehammer Symbols
37.33 (1.192/2430) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 102/1310MB 12:56 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:56:27

```

```

Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
22 where
23   "join Leaf Leaf = T1 Leaf"
24   "join (Node2 t1 a t2) (Node2 t3 b t4) = (
25     case (join t2 t3) of
26       T1 t23 ⇒ T1 (Node3 t1 a t23 b t4)
27     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29   "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30     case (join t2 t3) of
31       T1 t23 ⇒ Upi (Node2 t1 a t23) b (Node2 t4 c t5)
32     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33   )"
34   "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35     case (join t3 t4) of
36       T1 t34 ⇒ Upi (Node2 t1 a t2) b (Node2 t34 c t5)
37     | Upi t3' x t4' ⇒ Upi (Node2 t1 a t2) b (Node3 t3' b t4 c t5)
38   )"
39
40
41
42
Output Query Sledgehammer Symbols
37.55 (1.214/2429) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 99/1310MB 12:56 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy (modified) 12:56:56

```

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
27   | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node2 t3' b t4)
28   )"
29   "join (Node2 t1 a t2) (Node3 t3 b t4 c t5) = (
30     case (join t2 t3) of
31       T1 t23 ⇒ Upi (Node2 t1 a t23) b (Node2 t4 c t5)
32     | Upi t2' x t3' ⇒ Upi (Node2 t1 a t2') x (Node3 t3' b t4 c t5)
33   )"
34   "join (Node3 t1 a t2 b t3) (Node2 t4 c t5) = (
35     case (join t3 t4) of
36       T1 t34 ⇒ Upi (Node2 t1 a t2) b (Node2 t34 c t5)
37     | Upi t3' x t4' ⇒ Upi (Node2 t1 a t2) b (Node3 t3' x t4' c t5)
38   )"
39   "join (Node3 t1 a t2 b t3) (Node3 t4 c t5 d t6) = (
40     case (join t3 t4) of
41       T1 t34 ⇒ Upi (Node2 t1 a t2) b (Node2 t34 c t5)
42     | Upi t3' x t4' ⇒ Upi (Node2 t1 a t2) b (Node3 t3' x t4' c t5)
43   )"
44
45
46
47
Output Query Sledgehammer Symbols
40.21 (1.306/2638) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 71/1310MB 12:58 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 12:58:00

```



```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
44 | "join _ _ = T1 Leaf"
45
46
47
48 lemma join_inorder:
49   fixes t1 t2 :: "nat tree23"
50   assumes "height t1 = height t2"
51   assumes "bal t1" "bal t2"
52   shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
53   quickcheck
54   oops
55
56 lemma join_bal:
57   fixes t1 t2 :: "'a tree23"
58   assumes "height t1 = height t2"
59   assumes "bal t1" "bal t2"
60   shows "bal (treei (join t1 t2)) = height (treei (join t1 t2))"
61   oops
62
proof (prove)
goal (1 subgoal):
1. inorder (treei (join t1 t2)) = inorder t1 @ inorder t2

```

```

Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
46 lemma join_inorder:
47   fixes t1 t2 :: "nat tree23"
48   assumes "height t1 = height t2"
49   assumes "bal t1" "bal t2"
50   shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
51   quickcheck
52   oops
53
54 lemma join_bal:
55   fixes t1 t2 :: "'a tree23"
56   assumes "height t1 = height t2"
57   assumes "bal t1" "bal t2"
58   shows "bal (treei (join t1 t2)) = height (treei (join t1 t2))"
59   oops
60
proof (prove)
goal (1 subgoal):
1. inorder (treei (join t1 t2)) = inorder t1 @ inorder t2

```

```

Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
51   assumes "bal t1" "bal t2"
52   shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
53   apply (induction t1 t2 rule: join.induct)
54   apply (auto split: up)
55
56
57 lemma join_bal:
58   fixes t1 t2 :: "'a tree23"
59   assumes "height t1 = height t2"
60   assumes "bal t1" "bal t2"
61   shows "bal (treei (join t1 t2)) = height (treei (join t1 t2))"
62   oops
63
proof (prove)
goal (8 subgoals):
1.  $\forall t1\ a\ t2\ t3\ b\ t4.$ 
   inorder (treei (join t2 t3)) = inorder t2 @ inorder t3  $\implies$ 
   inorder
   (treei
     (case join t2 t3 of T1 t23  $\implies$  T1 (t1, a, t23, b, t4)
      | Up1 t2' x t3'  $\implies$  Up1 (t1, a, t2') x (t3', b, t4))) =
   inorder t1 @ a # inorder t2 @ inorder t3 @ b # inorder t4
2.  $\forall t1\ a\ t2\ t3\ b\ t4\ c\ t5.$ 

```

```

Isabelle2017 - ex08tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
51   assumes "bal t1" "bal t2"
52   shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
53   apply (induction t1 t2 rule: join.induct)
54   apply (auto split: upz)
55
56
57 lemma join_bal:
58   fixes t1 t2 :: "'a tree23"
59   assumes "height t1 = height t2"
60   assumes "bal t1" "bal t2"
61   shows "bal (treei (join t1 t2)) = height (treei (join t1 t2))"
62   oops
63
proof (prove)
goal (8 subgoals):
1.  $\forall t1\ a\ t2\ t3\ b\ t4.$ 
   inorder (treei (join t2 t3)) = inorder t2 @ inorder t3  $\implies$ 
   inorder
   (treei
     (case join t2 t3 of T1 t23  $\implies$  T1 (t1, a, t23, b, t4)
      | Up1 t2' x t3'  $\implies$  Up1 (t1, a, t2') x (t3', b, t4))) =
   inorder t1 @ a # inorder t2 @ inorder t3 @ b # inorder t4
2.  $\forall t1\ a\ t2\ t3\ b\ t4\ c\ t5.$ 

```



```

Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
49 fixes t1 t2 :: "nat tree23"
50 assumes "height t1 = height t2"
51 assumes "bal t1" "bal t2"
52 shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
53 apply (induction t1 t2 rule: join.induct)
54 apply (simp_all split: up1.splits)
55
56
proof (prove)
goal (1 subgoal):
1. inorder (treei (join t1 t2)) = inorder t1 @ inorder t2

```

52.8 (1595/2748) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 897/1275MB 1:09 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 13:09:23

```

Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
49 fixes t1 t2 :: "nat tree23"
50 assumes "height t1 = height t2"
51 assumes "bal t1" "bal t2"
52 shows "inorder (treei (join t1 t2)) = (inorder t1 @ inorder t2)"
53 using assms
54 apply (induction t1 t2 rule: join.induct)
55 apply (auto split: up1.splits)
56 done

```

52.7 (1594/2764) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 897/1249MB 1:10 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 13:10:09

```

Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
57
58
59 lemma join_bal:
60 fixes t1 t2 :: "'a tree23"
61 assumes "height t1 = height t2"
62 assumes "bal t1" "bal t2"
63 shows "bal (treei (join t1 t2)) ^ height (join t1 t2) = height t2"
64oops
65
66 text <Hints:
67   ■ Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar
68   (There will be dozens of cases).
69   ■ To find bugs in your join function, or isolate the case where your automatic proof does not (yet) work

```

64.1 (1934/2764) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 897/1249MB 1:10 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 13:10:33

```

Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
56 done
57
58 lemma join_bal:
59 fixes t1 t2 :: "'a tree23"
60 assumes "height t1 = height t2"
61 assumes "bal t1" "bal t2"
62 shows "bal (treei (join t1 t2)) ^ height (join t1 t2) = height t2"
63 using assms
64 apply (induction t1 t2 rule: join.induct)
65 apply (auto split: up1.splits)
66 done
67

```

63.9 (1872/2856) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 897/1249MB 1:10 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmplthy 13:10:57

Isabelle2017 - ex08tmplthy

```

56 done
57
58
59 lemma join_bal:
60   fixes t1 t2 :: "'a tree23"
61   assumes "height t1 = height t2"
62   assumes "bal t1" "bal t2"
63   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
64   using assms
65   apply (induction t1 t2 rule: join.induct)
66   apply (auto split: upi.splits)
67   done

```

proof (prove)
goal (1 subgoal):
1. bal (tree_i (join t1 t2)) ∧ height (join t1 t2) = height t2

63.44 (1.907/2856) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 342/249MB 1:11 PM

Isabelle2017 - ex08tmplthy

```

56 done
57
58
59 lemma join_bal:
60   fixes t1 t2 :: "'a tree23"
61   assumes "height t1 = height t2"
62   assumes "bal t1" "bal t2"
63   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
64   using assms
65   apply (induction t1 t2 rule: join.induct)
66   apply (auto split: upi.splits)
67   done

```

proof (prove)
goal (1 subgoal):
1. bal (tree_i (join t1 t2)) ∧ height (join t1 t2) = height t2

63.23 (1.886/2856) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 368/249MB 1:11 PM

Isabelle2017 - ex08tmplthy

```

58
59 lemma join_bal:
60   fixes t1 t2 :: "'a tree23"
61   assumes "height t1 = height t2"
62   assumes "bal t1" "bal t2"
63   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
64   using assms
65   apply (induction t1 t2 rule: join.induct)
66   apply (auto split: upi.splits)
67   done
68
69 text <Hints:
70   Try to use automatic case splitting (auto split: upi.splits) instead of explicit case splitting via case.

```

proof (prove)
goal (1 subgoal):
1. bal (tree_i (join t1 t2)) ∧ height (join t1 t2) = height t2

63.28 (1.891/2856) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 89/225MB 1:12 PM

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```

58
59 lemma join_bal:
60   fixes t1 t2 :: "'a tree23"
61   assumes "height t1 = height t2"
62   assumes "bal t1" "bal t2"
63   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
64   using assms
65   apply (induction t1 t2 rule: join.induct)
66   apply (auto split: upi.splits)
67   done
68
69 text <Hints:
70   Try to use automatic case splitting (auto split: upi.splits) instead of explicit case splitting via case.

```

proof (prove)
goal (1 subgoal):
1. bal (tree_i (join t1 t2)) ∧ height (join t1 t2) = height t2

63.69 (1.932/2856) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 32/225MB 1:13 PM

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64.14 (1947/2856)
(Isabelle, Isabelle, UTF-8 Isabelle) tmr o UG 147/1.225MB 1:13 PM
Isabelle2017 - ex08tmpl.thy

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```

58 lemma join_bal:
59   fixes t1 t2 :: "'a tree23"
60   assumes "height t1 = height t2"
61   assumes "bal t1" "bal t2"
62   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
63   using assms
64   apply (induction t1 t2 rule: join.induct)
65   apply (auto split: up1.splits)
66   done
67
68 text <Hints:
69   Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar

```

```

proof (prove)
goal (1 subgoal):
1. bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2

```

```

66.1 (1992/2856)
(Isabelle, Isabelle, UTF-8 Isabelle) tmr o UG 161/1.225MB 1:13 PM
Isabelle2017 - ex08tmpl.thy

```

```

58 lemma join_bal:
59   fixes t1 t2 :: "'a tree23"
60   assumes "height t1 = height t2"
61   assumes "bal t1" "bal t2"
62   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
63   using assms
64   apply (induction t1 t2 rule: join.induct)
65   apply (auto split: up1.splits)
66   done
67
68 text <Hints:
69   Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar

```

```

proof (prove)
goal (9 subgoals):
1. [height () = height (); bal (); bal ()] ⇒ bal (treei (join () ())) ∧ height (join () ()) = height ()
2. ∧ t1 a t2 t3 b t4.
   [[height t2 = height t3; bal t2; bal t3]
   ⇒ bal (treei (join t2 t3)) ∧ height (join t2 t3) = height t3;

```

```

66.1 (1992/2856)
(Isabelle, Isabelle, UTF-8 Isabelle) tmr o UG 174/1.225MB 1:14 PM
Isabelle2017 - ex08tmpl.thy

```

```

58 lemma join_bal:
59   fixes t1 t2 :: "'a tree23"
60   assumes "height t1 = height t2"
61   assumes "bal t1" "bal t2"
62   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
63   using assms
64   apply (induction t1 t2 rule: join.induct)
65   apply (auto split: up1.splits)
66   done
67
68 text <Hints:
69   Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar

```

```

proof (prove)
goal (9 subgoals):
1. [height () = height (); bal (); bal ()] ⇒ bal (treei (join () ())) ∧ height (join () ()) = height ()
2. ∧ t1 a t2 t3 b t4.
   [[height t2 = height t3; bal t2; bal t3]
   ⇒ bal (treei (join t2 t3)) ∧ height (join t2 t3) = height t3;

```

```

66.43 (2034/2881)
(Isabelle, Isabelle, UTF-8 Isabelle) tmr o UG 146/1.225MB 1:14 PM
Isabelle2017 - ex08tmpl.thy

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```

58 lemma join_bal:
59   fixes t1 t2 :: "'a tree23"
60   assumes "height t1 = height t2"
61   assumes "bal t1" "bal t2"
62   shows "bal (treei (join t1 t2)) ∧ height (join t1 t2) = height t2"
63   using assms
64   apply (induction t1 t2 rule: join.induct)
65   apply (auto split: up1.splits) (* Takes roughly 30sec*)
66   done
67
68 text <Hints:
69   Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar

```

```

proof (prove)
goal:
No subgoals!

```

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```

60 fixes t1 t2 :: "'a tree23"
61 assumes "height t1 = height t2"
62 assumes "bal t1" "bal t2"
63 shows "bal (treei (join t1 t2)) ^ height (join t1 t2) = height t2"
64 using assms
65 apply (induction t1 t2 rule: join.induct)
66 apply (auto split!: up1.splits)
67 done
68
69 text <Hints:
70   ■ Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar
71   (There will be dozens of cases).
72   ■ To find bugs in your join function, or isolate the case where your automatic proof does not (yet) work

```

proof (prove)
goal:
No subgoals!

66.20 (2011/2857) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 590.1 225MB 1:15 PM

Isabelle2017 - ex08tmplthy

```

60 fixes t1 t2 :: "'a tree23"
61 assumes "height t1 = height t2"
62 assumes "bal t1" "bal t2"
63 shows "bal (treei (join t1 t2)) ^ height (join t1 t2) = height t2"
64 using assms
65 apply (induction t1 t2 rule: join.induct)
66 apply (auto split!: up1.splits)
67 done
68
69 text <Hints:
70   ■ Try to use automatic case splitting (<auto split: ...>) instead of explicit case splitting via Isar
71   (There will be dozens of cases).
72   ■ To find bugs in your join function, or isolate the case where your automatic proof does not (yet) work

```

proof (prove)
goal:
No subgoals!

66.21 (2012/2857) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 604.1 225MB 1:16 PM

Isabelle2017 - ex08tmplthy

```

76 text <
77   \Exercise{Bounding the Size of 2-3-Trees}
78
79   Show that for 2-3-trees, we have:
80   @{\text [display] < log3 ( s(t) + 1 ) ≤ h(t) ≤ log2 ( s(t) + 1 )>}
81
82   Hint: It helps to first raise the two sides of the
83   inequation to the 2nd/3rd power.
84   Use sledgehammer and find-theorems to search for the appropriate lemmas.
85
86
87 lemma height_bound_upper: "bal t ⇒ height t ≤ log 2 (size t + 1)"

```

80.13 (2459/2857) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 620.1 25MB 1:16 PM

Isabelle2017 - ex08tmplthy

```

76 text <
77   \Exercise{Bounding the Size of 2-3-Trees}
78
79   Show that for 2-3-trees, we have:
80   @{\text [display] < log3 ( s(t) + 1 ) ≤ h(t) ≤ log2 ( s(t) + 1 )>}
81
82   Hint: It helps to first raise the two sides of the
83   inequation to the 2nd/3rd power.
84   Use sledgehammer and find-theorems to search for the appropriate lemmas.
85
86
87 lemma height_bound_upper: "bal t ⇒ height t ≤ log 2 (size t + 1)"

```

80.34 (2479/2857) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 630.1 25MB 1:17 PM

```

Isabelle2017 - ex08tmpl.thy
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ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
76 text <
77 \Exercise{Bounding the Size of 2-3-Trees}
78
79 Show that for 2-3-trees, we have:
80 
$$\log_3 (s(t) + 1) \leq h(t) \leq \log_2 (s(t) + 1)$$

81
82 Hint: It helps to first raise the two sides of the
83 inequation to the 2nd/3rd power.
84 Use sledgehammer and find-theorems to search for the appropriate lemmas.
85
86
87 Lemma height_bound_upper: "bal t  $\implies$  height t  $\leq$  log 2 (size t + 1)"
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Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
91 from assms have "2^height t ≤ size t + 1"
92   by (induction t) auto
93   then show ?thesis
94     using le_log2_of_power by blast
95 qed
96
97 lemma height_bound_lower: "bal t ⇒ log 3 (size t + 1) ≤ height t"
98   oops
99
100 (*<*)
101 end
102 (*>*)

proof (prove)
goal (1 subgoal):
1. bal t ⇒ log 3 (real (size t + 1)) ≤ real (height t)

```

97.49 (2964/3006) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 861.1 303MB 1:22 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 13:22:41

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Isabelle2017 - ex08tmpl.thy
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ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
92   by (induction t) auto
93   then show ?thesis
94     using le_log2_of_power by blast
95 qed
96
97 lemma height_bound_lower:
98   assumes "bal t"
99   shows "log 3 (size t + 1) ≤ height t"
100   oops
101
102 (*<*)
103 end
104 (*>*)

theorem height_bound_upper:
bal ?t ⇒ real (height ?t) ≤ log 2 (real (size ?t + 1))

```

96.1 (2915/3024) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 842.1 303MB 1:28 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 13:28:27

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Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
95 qed
96
97 thm log_of_power_le[where b=3, simplified]
98 lemma height_bound_lower:
99   assumes "bal t"
100   shows "log 3 (size t + 1) ≤ height t"
101   oops
102
103 (*<*)
104 end
105 (*>*)
106

[?m ≤ 3 ^ ?n; 0 < ?m] ⇒ log 3 (real ?m) ≤ real ?n

```

103.1 (3051/3067) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 877.1 304MB 1:29 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... ex08.pdf Isabelle2017 - ex08tmpl.thy 13:29:46

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Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
91 from assms have "2^height t ≤ size t + 1"
92   by (induction t) auto
93   then show ?thesis
94     using le_log2_of_power by blast
95 qed
96
97 thm log_of_power_le
98
99 lemmas log3_of_power_le = log_of_power_le[where b=3, simplified]
100
101 lemma height_bound_lower:
102   assumes "bal t"
103   shows "log 3 (size t + 1) ≤ height t"

real ?m ≤ ?b ^ ?n; 1 < ?b; 0 < ?m] ⇒ log ?b (real ?m) ≤ real ?n

```

97.5 (2920/3111) null parsing complete, 0 error(s) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 877.1 302MB 1:30 PM
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Isabelle2017 - ex08tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
93 then show ?thesis
94 using le_log2_of_power by blast
95 qed
96
97 Lemmas log3_of_power_le = log_of_power_le[where b=3, simplified]
98
99 Lemma height_bound_lower:
100 assumes "bal t"
101 shows "log 3 (size t + 1) ≤ height t"
102 proof
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Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
92 by (induction t) auto
93 then show ?thesis
94 using le_log2_of_power by blast
95 qed
96
97 lemmas log3_of_power_le = log_of_power_le[where b=3, simplified]
98
99 Lemma height_bound_lower:
100 assumes "bal t"
101 shows "log 3 (size t + 1) ≤ height t"
102 proof -
103 from assms have "size t + 1 ≤ 3^height t"
104 by (induction t) auto
105 then show ?thesis
106 using log3_of_power_le by fastforce

proof (prove)
goal (1 subgoal):
1. log 3 (real (size t + 1)) ≤ real (height t)

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Isabelle2017 - ex08tmplthy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
99 Lemma height_bound_lower:
100 assumes "bal t"
101 shows "log 3 (size t + 1) ≤ height t"
102 proof -
103 from assms have "size t + 1 ≤ 3^height t"
104 by (induction t) auto
105 then show ?thesis
106
107 apply (fastforce+) [4].
108 using log3_of_power_le by fastforce
109 qed
110
111 (*<*)
112 end
113 (*>*)

proof (prove)
using this:
size t + 1 ≤ 3 ^ height t

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Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
99 Lemma height_bound_lower:
100 assumes "bal t"
101 shows "log 3 (size t + 1) ≤ height t"
102 proof -
103 from assms have "size t + 1 ≤ 3^height t"
104 by (induction t) auto
105 then show ?thesis
106 apply (fastforce+) [4]
107 using log3_of_power_le by fastforce
108 qed
109
110 (*<*)
111 end
112 (*>*)

proof (prove)
using this:
size t + 1 ≤ 3 ^ height t

```

```

Isabelle2017 - ex08tmplthy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
99 Lemma height_bound_lower:
100 assumes "bal t"
101 shows "log 3 (size t + 1) ≤ height t"
102 proof -
103 from assms have "size t + 1 ≤ 3^height t"
104 by (induction t) auto
105 then show ?thesis
106 using log3_of_power_le by fastforce
107 qed
108
109 (*<*)
110 end
111 (*>*)

have size t + 1 ≤ 3 ^ height t
proof (state)
this:
size t + 1 ≤ 3 ^ height t

```

```
Isabelle2017 - ex08tmplthy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
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101   shows "log 3 (size t + 1) ≤ height t"
102   proof -
103     from assms have "size t + 1 ≤ 3^height t"
104     apply (auto, fastforce)
105     ..
106     by (induction t) auto
107     then show ?thesis
108     using log3_of_power_le by fastforce
109   qed
110
111 (*<*)
112 end
113 (*>*)

proof (prove)
using this:
bal t
```

```
Isabelle2017 - ex08tmplthy (modified)
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using this:
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101   shows "log 3 (size t + 1) ≤ height t"
102   proof -
103     from assms have "size t + 1 ≤ 3^height t"
104     apply (auto, (linarith | fastforce))
105     ..
106     by (induction t) auto
107     then show ?thesis
108     using log3_of_power_le by fastforce
109   qed
110
111 (*<*)
112 end
113 (*>*)

proof (prove)
using this:
bal t
```

Isabelle2017 - ex08tmplthy

```

File Edit Search Markers Folding View Utilities Macros Plugins Help
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101   shows "log 3 (size t + 1) ≤ height t"
102   proof -
103     from assms have "size t + 1 ≤ 3^height t"
104     apply (auto; (linarith))
105
106     by (induction t) auto
107     then show ?thesis
108     using log3_of_power_le by fastforce
109   qed
110
111 (*+)
112 end
113 (*+)

```

Proof state Auto update Update Search: 100%

```

proof (prove)
goal (1 subgoal):
1. bal t ⇒ Suc (size t) ≤ 3 ^ height t

```

Output Query Sledgehammer Symbols

104,27 (3144/3256) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1:266MB 1:42 PM

Isabelle2017 - ex08tmplthy

```

File Edit Search Markers Folding View Utilities Macros Plugins Help
ex08tmplthy (~/lehre/FDS/SS18/public/exercises/)
99 Lemma height_bound_lower:
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112 end
113 (*+)

```

Proof state Auto update Update Search: 100%

```

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goal (1 subgoal):
1. bal t ⇒ Suc (size t) ≤ 3 ^ height t

```

Output Query Sledgehammer Symbols

104,27 (3144/3256) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1:266MB 1:42 PM

Examples

- src/HOL/ex/Seq.thy
- src/HOL/ex/ML.thy
- src/HOL/Univ/Univ.thy
- src/HOL/Isar_Examples/Drinker.thy
- src/Tools/SMLExamples.thy

Release notes

- ANNOUNCE
- README
- NEWS
- COPYRIGHT
- CONTRIBUTORS
- contrib/README
- src/Tools/Edit/README

Tutorials

- prog-prove: Programming and Proving in Isabelle/HOL
- locales: Tutorial on Locales
- classes: Tutorial on Type Classes
- datatypes: Tutorial on (Co)datatype Definitions
- functions: Tutorial on Function Definitions
- corec: Tutorial on Nonprimatively Corecursive Definitions
- codegen: Tutorial on Code Generation
- nitpick: User's Guide to Nitpick
- sledgehammer: User's Guide to Sledgehammer
- eisbach: The Eisbach User Manual
- sugan: LaTeX Sugar for Isabelle documents

Reference Manuals

- main: What's in Main
- Isar-ref: The Isabelle/Isar Reference Manual

lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises

```

File Edit View Search Terminal Help
lammich@lapnikow10 ~/opt/ttt$ cd ~/Lehre/FDS/SS18/public/exercises/
lammich@lapnikow10 ~/Lehre/FDS/SS18/public/exercises$ ls
ex01.pdf ex07.pdf tmp103.thy- tut01.thy tut04.thy- tut07.thy-
ex02.pdf ex08.pdf tmp104.thy- tut02.thy tut05.thy-
ex03.pdf ex08tmpl.thy tmp104.thy- tut02.thy tut05.thy-
ex04.pdf hw08bonustmpl.thy tmp105.thy- tut03.thy tut06.thy-
ex05.pdf hw08tmpl.thy tmp106.thy- tut03.thy- tut06.thy-
ex06.pdf tmp103.thy tmp107.thy- tut04.thy tut07.thy
lammich@lapnikow10 ~/Lehre/FDS/SS18/public/exercises$ evince ex08.pdf
^Z
[1]+ Stopped evince ex08.pdf
lammich@lapnikow10 ~/Lehre/FDS/SS18/public/exercises$ bg
[1]+ evince ex08.pdf &
lammich@lapnikow10 ~/Lehre/FDS/SS18/public/exercises$ isabelle_jedit ex08tmpl.thy

```

```

113 (*+)

```

Proof state Auto update Update Search: 100%

```

have size t + 1 ≤ 3 ^ height t
proof (state)
this:
size t + 1 < 3 ^ height t

```

Output Query Sledgehammer Symbols

106,26 (3174/3256) Input/output complete (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1:266MB 1:43 PM

lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises

```

File Edit View Search Terminal Help
HG Enter commit message. Lines beginning with 'HG:' are removed
HG Leave message empty to abort commit
HG -
HG user: lammich@lapnikow10:~/lehre/FDS/SS18/public/exercises
HG branch: default
HG changed exercises/ex08tmpl.thy

```

```

113 (*+)

```

Proof state Auto update Update Search: 100%

```

have size t + 1 ≤ 3 ^ height t
proof (state)
this:
size t + 1 < 3 ^ height t

```

Output Query Sledgehammer Symbols

106,26 (3174/3256) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1:266MB 1:43 PM

