

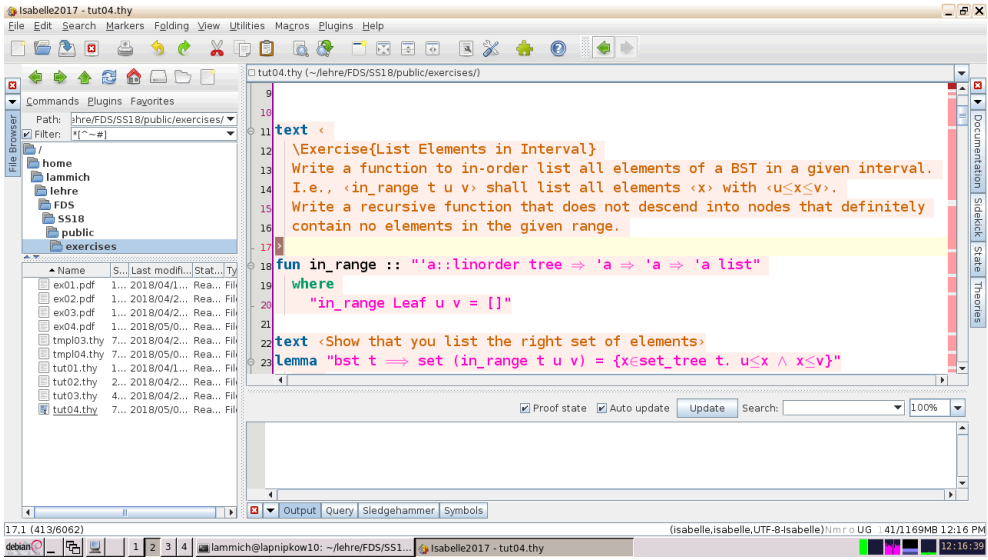
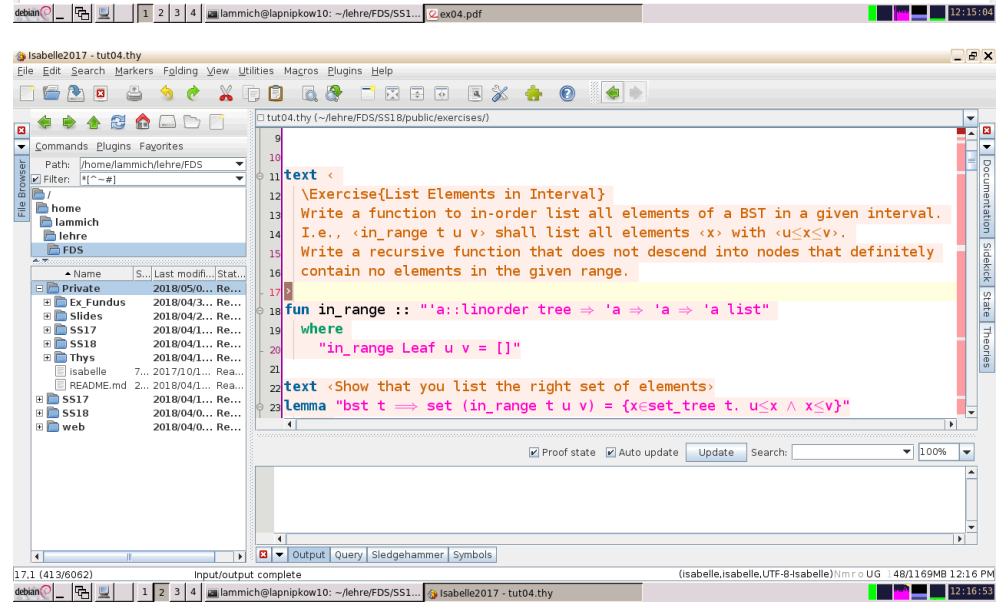
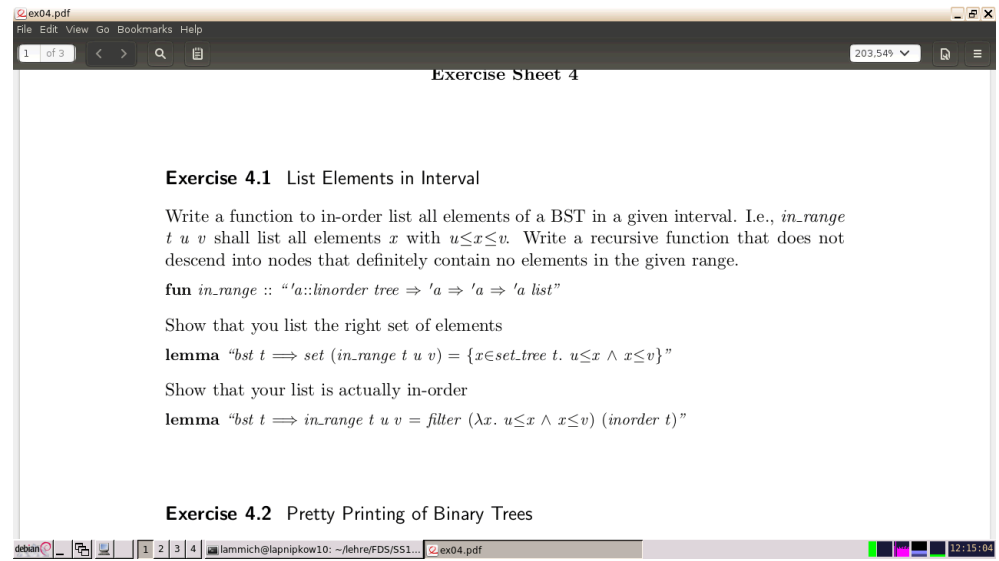
# Script generated by TTT

Title: Lammich: FDS Tutorial (04.05.2018)

Date: Fri May 04 12:15:04 CEST 2018

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Pages: 108



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Isabelle2017 - ex03.thy
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ex03.thy (~/.lehre/FDS/Private/SS18/Exercises/ex03/)
270 You may want to start with an auxiliary function, that takes the BST with
271 the elements seen so far as additional argument, and then define the actual function.
272 >
273
274 fun bst_remdups_aux :: "'a::linorder tree => 'a list => 'a list"
275 (*<+*) where
276   "bst_remdups_aux _ [] = []"
277 | "bst_remdups_aux t (x#xs) = (if isin t x then bst_remdups_aux t xs else x#bst_remdups_aux (ins x t) xs)"
278 (*>+*)
279
280 definition "bst_remdups xs ≡ bst_remdups_aux Leaf xs"
281
282
283 text <Show that your function preserves the set of elements, and returns a list
284 with no duplicates (predicate <distinct> in Isabelle).
285 Hint: Generalization!
286 >
287 (*<+*)
288 Lemma bst_remdups_aux_set[simp]: "bst t ==> set (bst_remdups_aux t xs) = set xs - set_tree t"
289 apply (induction xs arbitrary: t)
290 apply (auto simp: set_tree_isin set_tree_ins bst_ins)
291 done
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```

Exercise Sheet 4

### Exercise 4.1 List Elements in Interval

Write a function to in-order list all elements of a BST in a given interval. I.e.,  $\text{in\_range } t \ u \ v$  shall list all elements  $x$  with  $u \leq x \leq v$ . Write a recursive function that does not descend into nodes that definitely contain no elements in the given range.

```
fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
```

Show that you list the right set of elements

```
lemma "bst t => set (in_range t u v) = {x∈set_tree t. u≤x ∧ x≤v}"
```

Show that your list is actually in-order

```
lemma "bst t => in_range t u v = filter (λx. u≤x ∧ x≤v) (inorder t)"
```

### Exercise 4.2 Pretty Printing of Binary Trees

```

9
10
11 text <
12 \Exercise(List Elements in Interval)
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u≤x≤v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19   where
20     "in_range Leaf u v = []"
21
22 text <Show that you list the right set of elements>
23 lemma "bst t => set (in_range t u v) = {x∈set_tree t. u≤x ∧ x≤v}"
24   oops
25
26 text <Show that your list is actually in-order>
27 lemma "bst t => in_range t u v = filter (λx. u≤x ∧ x≤v) (inorder t)"
28   oops
29

```

```

10
11 text <
12 \Exercise(List Elements in Interval)
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u≤x≤v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19   where
20     "in_range Leaf u v = []"
21
22 text <Show that you list the right set of elements>
23 lemma "bst t => set (in_range t u v) = {x∈set_tree t. u≤x ∧ x≤v}"
24   oops
25
26 text <Show that your list is actually in-order>
27 lemma "bst t => in_range t u v = filter (λx. u≤x ∧ x≤v) (inorder t)"
28   oops
29

```

```

10
11 text <
12 \Exercise(List Elements in Interval)
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u≤x≤v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19   where
20     "in_range Leaf u v = []"
21
22 text <Show that you list the right set of elements>
23 lemma "bst t => set (in_range t u v) = {x∈set_tree t. u≤x ∧ x≤v}"
24   oops
25
26 text <Show that your list is actually in-order>
27 lemma "bst t => in_range t u v = filter (λx. u≤x ∧ x≤v) (inorder t)"
28   oops
29

```

```
Isabelle2017 - tut04.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
10
11 \Exercise(List Elements in Interval)
12 Write a function to in-order list all elements of a BST in a given interval.
13 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
14 Write a recursive function that does not descend into nodes that definitely
15 contain no elements in the given range.
16
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v = "
22
23 text <Show that you list the right set of elements>
24 lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
25   oops
26
27 text <Show that your list is actually in-order>
28 lemma "bst t => in_range t u v = filter (λx. u<=x & x<=v) (inorder t)"
29   oops
30
Output Query Sledgehammer Symbols
21.24 (536/6102) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 31/2/1188MB 12:27 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy (modified) 12:27:28
```

```
Isabelle2017 - tut04.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
10
11 \Exercise(List Elements in Interval)
12 Write a function to in-order list all elements of a BST in a given interval.
13 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
14 Write a recursive function that does not descend into nodes that definitely
15 contain no elements in the given range.
16
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if a>u then #in_range l u v else [])
23   @ (if a<=v then #in_range r u v else [])"
24
25 text <Show that you list the right set of elements>
26 lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
27   oops
28
29 text <Show that your list is actually in-order>
30
Output Query Sledgehammer Symbols
23.22 (613/6178) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 62/1169MB 12:28 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy (modified) 12:28:43
```

```
Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
10
11 \Exercise(List Elements in Interval)
12 Write a function to in-order list all elements of a BST in a given interval.
13 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
14 Write a recursive function that does not descend into nodes that definitely
15 contain no elements in the given range.
16
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if a>u then #in_range l u v else [])
23   @ (if a<=v then #in_range r u v else [])"
24
25 text <Show that you list the right set of elements>
26 lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
27   oops
28
29 text <Show that your list is actually in-order>
30
Output Query Sledgehammer Symbols
22.21 (566/6198) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 44/1169MB 12:29 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:29:38
```

```
Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
10
11 \Exercise(List Elements in Interval)
12 Write a function to in-order list all elements of a BST in a given interval.
13 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
14 Write a recursive function that does not descend into nodes that definitely
15 contain no elements in the given range.
16
17
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if a>u then #in_range l u v else [])
23   @ (if a<=v then #in_range r u v else [])"
24
25 text <Show that you list the right set of elements>
26 lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
27   oops
28
29 text <Show that your list is actually in-order>
30
Output Query Sledgehammer Symbols
22.20 (565/6198) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 44/1169MB 12:30 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:30:19
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Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <
12 \Exercise{List Elements in Interval}
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17 >
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if a >= u then #in_range l u v else [])
23   @ (if a <= v then #in_range r u v else [])"
24
25
26 text <Show that you list the right set of elements>
27 Lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
28
29
30 text <Show that your list is actually in-order>
31 Lemma "bst t => in_range t u v = filter (\x. u<=x & x<=v) (inorder t)"
32
Output Query Sledgehammer Symbols
21.15 (527/6198) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 599/1.69MB 12:31 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:31:06
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```
Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <
12 \Exercise{List Elements in Interval}
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17 >
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if must_descent_to l then in_range l u v else [])
23   @ (if must_take a then [a] else [])
24   @ (if mujst_descent_to r then in_range r u v else [])"
25
26
27 text <Show that you list the right set of elements>
28 Lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
29
30
31 text <Show that your list is actually in-order>
32
Output Query Sledgehammer Symbols
23.27 (629/6262) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 612/1.88MB 12:32 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:32:24
```

```
Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <
12 \Exercise{List Elements in Interval}
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17 >
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if must_descent_to l then in_range l u v else [])
23   @ (if must_take a then [a] else [])
24   @ (if mujst_descent_to r then in_range r u v else [])"
25
26
27 text <Show that you list the right set of elements>
28 Lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
29
30
31 text <Show that your list is actually in-order>
32
Output Query Sledgehammer Symbols
24.1 (643/6263) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 709/1.88MB 12:33 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:33:00
```

```
Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <
12 \Exercise{List Elements in Interval}
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e., <in_range t u v> shall list all elements <x> with <u<=x<=v>.
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17 >
18 fun in_range :: "'a::linorder tree => 'a => 'a => 'a list"
19 where
20   "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if must_descent_to l then in_range l u v else [])
23   @ (if must_take a then [a] else [])
24   @ (if mujst_descent_to r then in_range r u v else [])"
25
26
27 text <Show that you list the right set of elements>
28 Lemma "bst t => set (in_range t u v) = {x<set_tree t. u<=x & x<=v}"
29
30
31 text <Show that your list is actually in-order>
32
Output Query Sledgehammer Symbols
22.10 (555/6263) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 718/1.88MB 12:34 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:34:10
```

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
11 text <
12 \Exercise{List Elements in Interval}
13 Write a function to in-order list all elements of a BST in a given interval.
14 I.e.,  $\text{in\_range } t \ u \ v$  shall list all elements  $x$  with  $u \leq x \leq v$ .
15 Write a recursive function that does not descend into nodes that definitely
16 contain no elements in the given range.
17 >
18 fun in_range :: "a::linorder tree  $\Rightarrow$  'a  $\Rightarrow$  'a  $\Rightarrow$  'a list"
19 where
20 "in_range Leaf u v = []"
21 | "in_range (Node l a r) u v =
22   (if u < a then in_range l u v else [])
23   @ (if u <= a  $\wedge$  a <= v then [a] else [])
24   @ (if a < v then in_range r u v else [])"
25
26
27 text <Show that you list the right set of elements>
28 Lemma "bst t  $\Rightarrow$  set (in_range t u v) = {x  $\in$  set_tree t.  $u \leq x \wedge x \leq v$ }"
29
30 oops
31
32 text <Show that your list is actually in-order>

```

24.1 (627/6232) (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 251/1188MB 12:35 PM  
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy 12:35:53

```

Isabelle2017 - tut04.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
20 where
21 "in_range Leaf u v = []"
22 | "in_range (Node l a r) u v =
23   (if u < a then in_range l u v else [])
24   @ (if u <= a  $\wedge$  a <= v then [a] else [])
25   @ (if a < v then in_range r u v else [])"
26
27
28 text <Show that you list the right set of elements>
29 Lemma "bst t  $\Rightarrow$  set (in_range t u v) = {x  $\in$  set_tree t.  $u \leq x \wedge x \leq v$ }"
30
31 apply
32 apply (keyword)
33 text <Show that your list is actually in-order>
34 Lemma "bst t  $\Rightarrow$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
35
36 oops

```

30.7 (803/6233) (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 829/1189MB 12:37 PM  
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy (modified) 12:37:15

```

Isabelle2017 - tut04.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
20 where
21 "in_range Leaf u v = []"
22 | "in_range (Node l a r) u v =
23   (if u < a then in_range l u v else [])
24   @ (if u <= a  $\wedge$  a <= v then [a] else [])
25   @ (if a < v then in_range r u v else [])"
26
27
28 text <Show that you list the right set of elements>
29 Lemma "bst t  $\Rightarrow$  set (in_range t u v) = {x  $\in$  set_tree t.  $u \leq x \wedge x \leq v$ }"
30
31 apply (induction t)
32
33 text <Show that your list is actually in-order>
34 Lemma "bst t  $\Rightarrow$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
35
36 oops

```

30.21 (817/6247) Input/output complete (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 119/1187MB 12:37 PM  
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy (modified) 12:37:31

```

Isabelle2017 - tut04.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
32 done
33
34 text <Show that your list is actually in-order>
35 Lemma "bst t  $\Rightarrow$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
36
37 apply (induction t)
38
39
40
41

```

37.3 (980/6286) Input/output complete (isabelle,isabelle,UTF-8-isabelle)Nm r o UG 821/1187MB 12:38 PM  
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut04.thy (modified) 12:38:31

Isabelle2017 - tut04.thy

```

done
text <Show that your list is actually in-order>
lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
apply (induction t)
apply simp
apply simp
sledgehammer

```

Sledgehammering...

"spass": Timed out  
"cvc4": Timed out  
"z3": Timed out  
"e": Timed out

39.15 (1.018/6324) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 257/1187MB 12:40 PM

Isabelle2017 - tut04.thy

```

done
text <Show that your list is actually in-order>
lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
apply (induction t)
apply simp
apply simp
sledgehammer

```

Sledgehammering...

"spass": Timed out  
"cvc4": Timed out  
"z3": Timed out  
"e": Timed out

39.3 (1.006/6324) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 300/1187MB 12:40 PM

Isabelle2017 - tut04.thy

```

done
text <Show that your list is actually in-order>
lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
apply (induction t)
apply simp
apply simp

```

$\implies (u = x2 \wedge x2 = v \implies [v] = [x \text{--} \text{inorder } t1 . v \leq x \wedge x \leq v] @ v \# [x \text{--} \text{inorder } t2 . v \leq x \wedge x \leq v]) \wedge ((u = x2 \implies x2 \neq v) \implies (u = x2 \wedge x2 \leq v \implies (x2 < v \implies x2 \# \text{in\_range } t2 \ x2 \ v = [x \text{--} \text{inorder } t1 . x2 \leq x \wedge x \leq v] @ x2 \# [x \text{--} \text{inorder } t2 . x2 \leq x \wedge x \leq v]) \wedge (\neg x2 < v \implies [x \text{--} \text{inorder } t1 . x2 \leq x \wedge x \leq v] = [] \wedge [x \text{--} \text{inorder } t2 . x2 \leq x \wedge x \leq v] = [])) \wedge ((u = x2 \implies \neg x2 \leq v) \implies (x2 < v \implies$

39.1 (1.004/6310) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 44/1187MB 12:40 PM

Isabelle2017 - tut04.thy

```

done
text <Show that your list is actually in-order>
lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
apply (induction t)
apply simp
apply simp

```

proof (prove)
goal (1 subgoal):
1.  $\wedge t1 \ x2 \ t2. [in\_range \ t1 \ u \ v = [x \text{--} \text{inorder } t1 . u \leq x \wedge x \leq v]; in\_range \ t2 \ u \ v = [x \text{--} \text{inorder } t2 . u \leq x \wedge x \leq v]; bst \ t1 \ \wedge \ bst \ t2 \ \wedge (\forall x \in \text{set\_tree } t1. x < x2) \ \wedge (\forall x \in \text{set\_tree } t2. x2 < x)] \implies (u = x2 \wedge x2 = v \implies [v] = [x \text{--} \text{inorder } t1 . v \leq x \wedge x \leq v] @ v \# [x \text{--} \text{inorder } t2 . v \leq x \wedge x \leq v]) \wedge ((u = x2 \implies x2 \neq v) \implies (u = x2 \wedge x2 \leq v \implies (x2 < v \implies x2 \# \text{in\_range } t2 \ x2 \ v =$

39.13 (1.003/6310) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 467/1187MB 12:41 PM

Isabelle2017 - tut04.thy

```

35 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
36   apply (induction t)
37   apply simp
38   apply cclar_simp

```

proof (prove)  
goal (1 subgoal):  
1.  $\wedge t1\ x2\ t2.$   
[in\_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];  
in\_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < x2$ ;  
 $\forall x \in \text{set\_tree } t2. x2 < x$ ]  
 $\implies (u = x2 \wedge x2 = v \implies$   
[v] = [x←inorder t1 . v ≤ x ∧ x ≤ v] @ v # [x←inorder t2 . v ≤ x ∧ x ≤ v])  $\wedge$   
(u = x2  $\implies$  x2  $\neq$  v)  $\implies$   
(u = x2  $\wedge$  x2 ≤ v  $\implies$   
(x2 < v  $\implies$   
x2 # in\_range t2 x2 v =

38.13 (1.003/6314) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 579/1187MB 12:42 PM

Isabelle2017 - tut04.thy

```

35 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
36   apply (induction t)
37   apply simp
38   apply cclar_simp

```

text <Exercise(Pretty Printing of Binary Trees)>

proof (prove)  
goal (1 subgoal):  
1.  $\wedge t1\ x2\ t2.$   
[in\_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];  
in\_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < x2$ ;  
 $\forall x \in \text{set\_tree } t2. x2 < x$ ]  
 $\implies (u = x2 \wedge x2 = v \implies$   
[v] = [x←inorder t1 . v ≤ x ∧ x ≤ v] @ v # [x←inorder t2 . v ≤ x ∧ x ≤ v])  $\wedge$

38.13 (1.003/6314) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 634/1187MB 12:43 PM

Isabelle2017 - tut04.thy

```

34 Lemma ""
35 text <Show that your list is actually in-order>
36 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
37   apply (induction t)
38   apply simp
39   apply cclar_simp

```

Inner syntax error: unexpected end of input  
Failed to parse prop

40.8 (996/6325) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 632/1188MB 12:44 PM

Isabelle2017 - tut04.thy

```

35 Lemma ""
36 text <Show that your list is actually in-order>
37 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
38   apply (induction t)
39   apply simp
40   apply cclar_simp

```

proof (prove)  
goal (1 subgoal):  
1.  $\wedge t1\ x2\ t2.$   
[in\_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];  
in\_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < x2$ ;  
 $\forall x \in \text{set\_tree } t2. x2 < x$ ]  
 $\implies (u = x2 \wedge x2 = v \implies$   
[v] = [x←inorder t1 . v ≤ x ∧ x ≤ v] @ v # [x←inorder t2 . v ≤ x ∧ x ≤ v])  $\wedge$

41.8 (1009/6325) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 633/1188MB 12:45 PM



Isabelle2017 - tut04.thy

```

35 Lemma "
36
37 text <Show that your list is actually in-order>
38 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
39 apply (induction t)
40 apply simp
41 apply cllarsimp
42
43
44

```

Inner syntax error: unexpected end of input  
Failed to parse prop

35.9 (849/6325) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 6421 88MB 12:45 PM

Isabelle2017 - tut04.thy (modified)

```

31 apply auto
32 done
33
34
35 Lemma "[v] = l1@v#l2  $\longleftrightarrow$  "
36
37
38 text <Show that your list is actually in-order>
39 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
40 apply (induction t)
41

```

Inner syntax error: unexpected end of input  
Failed to parse prop

35.21 (861/6345) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 109/1189MB 12:45 PM

Isabelle2017 - tut04.thy (modified)

```

31 apply auto
32 done
33
34
35 Lemma "[v] = l1@v#l2  $\longleftrightarrow$  l1=[]  $\wedge$  l2=[]"
36
37 apply auto
38 tr
39 translations (keyword)
40 try (keyword)
41 try0 (keyword) list is actually in-order>
42 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"

```

proof (prove)  
goal (2 subgoals):  
1. [v] = l1 @ v # l2  $\implies$  l1 = []  
2. [v] = l1 @ v # l2  $\implies$  l2 = []

37.5 (897/6374) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 729/1189MB 12:46 PM

Isabelle2017 - tut04.thy

```

32 done
33
34
35 thm Cons_eq_append_conv
36 Lemma "[v] = l1@v#l2  $\longleftrightarrow$  l1=[]  $\wedge$  l2=[]"
37 by (auto simp: Cons_eq_append_conv)
38
39
40 text <Show that your list is actually in-order>
41 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
42

```

( $?x \# ?xs = ?ys @ ?zs$ ) = ( $?ys = [] \wedge ?x \# ?xs = ?zs \vee (\exists ys'. ?x \# ys' = ?ys \wedge ?xs = ys' @ ?zs)$ )

35.24 (864/6418) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 729/1189MB 12:48 PM

Isabelle2017 - tut04.thy (modified)

```

33
34
35 thm Cons_eq_append_conv
36 Lemma [] = l1@v#l2  $\longleftrightarrow$  l1=[]  $\wedge$  l2=[]
37 by (auto simp: Cons_eq_append_conv)
38
39
40 text <Show that your list is actually in-order>
41 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda$ x. u  $\leq$  x  $\wedge$  x  $\leq$  v) (inorder t)"
42 apply (induction t)
43 apply simp

```

Outer syntax error: command expected,  
but keyword ] $\Delta$  was found

36.7 (871/6417) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 293/1144MB 12:48 PM

Isabelle2017 - tut04.thy

```

33
34
35 thm Cons_eq_append_conv
36 Lemma [simp]: "[v] = l1@v#l2  $\longleftrightarrow$  l1=[]  $\wedge$  l2=[]"
37 apply auto
38
39
40 by (auto simp: Cons_eq_append_conv)
41
42 text <Show that your list is actually in-order>
43 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda$ x. u  $\leq$  x  $\wedge$  x  $\leq$  v) (inorder t)"

```

```

proof (prove)
goal (2 subgoals):
1. [v] = l1 @ v # l2  $\implies$  l1 = []
2. [v] = l1 @ v # l2  $\implies$  l2 = []

```

38.1 (925/6441) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 627/1144MB 12:50 PM

Isabelle2017 - tut04.thy

```

33
34
35 thm Cons_eq_append_conv
36 Lemma [simp]: "[v] = l1@v#l2  $\longleftrightarrow$  l1=[]  $\wedge$  l2=[]"
37 apply auto
38
39
40 by (auto simp: Cons_eq_append_conv)
41
42 text <Show that your list is actually in-order>
43 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda$ x. u  $\leq$  x  $\wedge$  x  $\leq$  v) (inorder t)"
44 apply (induction t)
45 apply simp
46 apply clarsimp

```

```

proof (prove)
goal (2 subgoals):
1. [v] = l1 @ v # l2  $\implies$  l1 = []
2. [v] = l1 @ v # l2  $\implies$  l2 = []

```

38.1 (925/6441) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 627/1144MB 12:50 PM

Isabelle2017 - tut04.thy

```

38
39
40 text <Show that your list is actually in-order>
41 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda$ x. u  $\leq$  x  $\wedge$  x  $\leq$  v) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply clarsimp
45
46

```

```

proof (prove)
goal (1 subgoal):
1.  $\wedge$ t1 x2 t2.
[in_range t1 u v = [x--inorder t1 . u  $\leq$  x  $\wedge$  x  $\leq$  v];
in_range t2 u v = [x--inorder t2 . u  $\leq$  x  $\wedge$  x  $\leq$  v]; bst t1; bst t2;  $\forall$ x $\in$ set_tree t1. x < x2;
 $\forall$ x $\in$ set_tree t2. x2 < x]
 $\implies$  (u = x2  $\wedge$  x2 = v  $\implies$ 
[x--inorder t1 . v  $\leq$  x  $\wedge$  x  $\leq$  v] = []  $\wedge$  [x--inorder t2 . v  $\leq$  x  $\wedge$  x  $\leq$  v] = [])  $\wedge$ 
((u = x2  $\implies$  x2  $\neq$  v)  $\implies$ 
(u = x2  $\implies$  x2  $\leq$  v))

```

44.17 (1119/6426) (isabelle,isabelle,UTF-8-isabelle)Nmr o UG 9/1116MB 12:51 PM

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
41 Lemma "bst t ==> in_range t u v = filter (λx. u < x ∧ x <= v) (inorder t)"
42   apply (induction t)
43   apply simp
44   apply clarsimp
45
46
47
48
49
proof (prove)
goal (1 subgoal):
1. ∧t1 x2 t2.
  [in_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];
  in_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2; ∀x∈set_tree t1. x < x2;
  ∀x∈set_tree t2. x2 < x]
  ==> (u = x2 ∧ x2 = v ==>
  [x←inorder t1 . v ≤ x ∧ x ≤ v] = [] ∧ [x←inorder t2 . v ≤ x ∧ x ≤ v] = []) ∧
  ((u = x2 ==> x2 ≠ v) ==>
  [x←inorder t1 . v ≤ x ∧ x ≤ v] = [] ∧ [x←inorder t2 . v ≤ x ∧ x ≤ v] = [])
  
```

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
41 Lemma "bst t ==> in_range t u v = filter (λx. u < x ∧ x <= v) (inorder t)"
42   apply (induction t)
43   apply simp
44   apply clarsimp
45
46
47
48
49
proof (prove)
goal (1 subgoal):
1. ∧t1 x2 t2.
  [in_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];
  in_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2; ∀x∈set_tree t1. x < x2;
  ∀x∈set_tree t2. x2 < x]
  ==> (u = x2 ∧ x2 = v ==>
  [x←inorder t1 . v ≤ x ∧ x ≤ v] = [] ∧ [x←inorder t2 . v ≤ x ∧ x ≤ v] = []) ∧
  ((u = x2 ==> x2 ≠ v) ==>
  [x←inorder t1 . v ≤ x ∧ x ≤ v] = [] ∧ [x←inorder t2 . v ≤ x ∧ x ≤ v] = [])
  
```

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
41 Lemma "bst t ==> in_range t u v = filter (λx. u < x ∧ x <= v) (inorder t)"
42   apply (induction t)
43   apply simp
44   apply clarsimp
45
46
47
48
49
proof (prove)
goal (1 subgoal):
1. ∧t1 x2 t2.
  [in_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];
  in_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2; ∀x∈set_tree t1. x < x2;
  ∀x∈set_tree t2. x2 < x]
  ==> (u = x2 ∧ x2 = v ==>
  [x←inorder t1 . v ≤ x ∧ x ≤ v] = [] ∧ [x←inorder t2 . v ≤ x ∧ x ≤ v] = []) ∧
  ((u = x2 ==> x2 ≠ v) ==>
  [x←inorder t1 . v ≤ x ∧ x ≤ v] = [] ∧ [x←inorder t2 . v ≤ x ∧ x ≤ v] = [])
  
```

```

Isabelle2017 - tut04.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
34 thm Cons_eq_append_conv
35 Lemma [simp]: "[v] = l1@v#l2 ==> l1=[] ∧ l2=[]"
36   by (auto simp: Cons_eq_append_conv)
37
38
39 Lemma "[x←xs. P x] = [] ==> ()".
40
41
42 text <Show that your list is actually in-order>
43 Lemma "bst t ==> in_range t u v = filter (λx. u < x ∧ x <= v) (inorder t)"

```

Inner syntax error: unexpected end of input  
Failed to parse prop

Isabelle2017 - tut04.thy

```

40 text <Show that your list is actually in-order>
41 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply simp

```

$$\begin{aligned}
&\implies (u = x2 \wedge x2 = v \implies (\forall x \in \text{set\_tree } t1. v \leq x \implies x \neq v) \wedge (\forall x \in \text{set\_tree } t2. v \leq x \implies x \neq v)) \wedge \\
&((u = x2 \implies x2 \neq v) \implies \\
&(u = x2 \wedge x2 \leq v \implies \\
&(x2 < v \implies \\
&x2 \# \text{in\_range } t2 \ x2 \ v = \\
&[x \leftarrow \text{inorder } t1. x2 \leq x \wedge x \leq v] @ x2 \# [x \leftarrow \text{inorder } t2. x2 \leq x \wedge x \leq v]) \wedge \\
&(\neg x2 < v \implies \\
&(\forall x \in \text{set\_tree } t1. x2 \leq x \implies \neg x \leq v) \wedge (\forall x \in \text{set\_tree } t2. x2 \leq x \implies \neg x \leq v))) \wedge \\
&(u = x2 \implies \neg x2 \leq v) \implies \\
&(x2 < v \implies \\
&\neg u < x2 \implies \\
&(u = x2 \wedge x2 \leq v \implies \\
&x2 \# \text{in\_range } t2 \ x2 \ v = \\
&[x \leftarrow \text{inorder } t1. x2 \leq x \wedge x \leq v] @ x2 \# [x \leftarrow \text{inorder } t2. x2 \leq x \wedge x \leq v]) \wedge
\end{aligned}$$

44.42 (1144/6453) (isabelle.isabelle,UTF-8-isabelle)Nmro UG 585/1153MB 12:55 PM

Isabelle2017 - tut04.thy (modified)

```

40 text <Show that your list is actually in-order>
41 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply (clarsimp simp: filter_empty_conv Cons_eq_append_conv)

```

$$\begin{aligned}
&(u = x2 \wedge x2 \leq v \implies \\
&(x2 < v \implies \\
&(\forall x \in \text{set\_tree } t1. x2 \leq x \implies \neg x \leq v) \wedge \text{in\_range } t2 \ x2 \ v = [x \leftarrow \text{inorder } t2. x2 \leq x \wedge x \leq v] \vee \\
&(\exists y'. x2 \# y' = [x \leftarrow \text{inorder } t1. x2 \leq x \wedge x \leq v] \wedge \\
&\text{in\_range } t2 \ x2 \ v = y' @ x2 \# [x \leftarrow \text{inorder } t2. x2 \leq x \wedge x \leq v])) \wedge \\
&(\neg x2 < v \implies \\
&(\forall x \in \text{set\_tree } t1. x2 \leq x \implies \neg x \leq v) \wedge (\forall x \in \text{set\_tree } t2. x2 \leq x \implies \neg x \leq v))) \wedge \\
&(u = x2 \implies \neg x2 \leq v) \implies \\
&(x2 < v \implies
\end{aligned}$$

44.60 (1162/6473) (isabelle.isabelle,UTF-8-isabelle)Nmro UG 595/1158MB 12:56 PM

Isabelle2017 - tut04.thy

```

40 text <Show that your list is actually in-order>
41 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply (clarsimp simp: filter_empty_conv)

```

$$\begin{aligned}
&\text{in\_range } t2 \ u \ v = [x \leftarrow \text{inorder } t2. u \leq x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < x2; \\
&\forall x \in \text{set\_tree } t2. x2 < x \\
&\implies (u = x2 \wedge x2 = v \implies (\forall x \in \text{set\_tree } t1. v \leq x \implies x \neq v) \wedge (\forall x \in \text{set\_tree } t2. v \leq x \implies x \neq v)) \wedge \\
&((u = x2 \implies x2 \neq v) \implies \\
&(u = x2 \wedge x2 \leq v \implies \\
&(x2 < v \implies \\
&x2 \# \text{in\_range } t2 \ x2 \ v = \\
&[x \leftarrow \text{inorder } t1. x2 \leq x \wedge x \leq v] @ x2 \# [x \leftarrow \text{inorder } t2. x2 \leq x \wedge x \leq v]) \wedge \\
&(\neg x2 < v \implies
\end{aligned}$$

44.42 (1144/6473) (isabelle.isabelle,UTF-8-isabelle)Nmro UG 595/1158MB 12:56 PM

Isabelle2017 - tut04.thy

```

40 text <Show that your list is actually in-order>
41 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply (clarsimp simp: filter_empty_conv)

```

$$\begin{aligned}
&\forall x \in \text{set\_tree } t2. x2 < x \\
&\implies (u = x2 \wedge x2 = v \implies (\forall x \in \text{set\_tree } t1. v \leq x \implies x \neq v) \wedge (\forall x \in \text{set\_tree } t2. v \leq x \implies x \neq v)) \wedge \\
&((u = x2 \implies x2 \neq v) \implies \\
&(u = x2 \wedge x2 \leq v \implies \\
&(x2 < v \implies \\
&x2 \# \text{in\_range } t2 \ x2 \ v = \\
&[x \leftarrow \text{inorder } t1. x2 \leq x \wedge x \leq v] @ x2 \# [x \leftarrow \text{inorder } t2. x2 \leq x \wedge x \leq v]) \wedge \\
&(\neg x2 < v \implies \\
&(\forall x \in \text{set\_tree } t1. x2 \leq x \implies \neg x < v) \wedge (\forall x \in \text{set\_tree } t2. x2 < x \implies \neg x < v))) \wedge
\end{aligned}$$

44.42 (1144/6473) (isabelle.isabelle,UTF-8-isabelle)Nmro UG 599/1158MB 12:57 PM

Isabelle2017 - tut04.thy

```

41 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
42   apply (induction t)
43   apply simp
44   apply (clarsimp simp: filter_empty_conv)

```

```

proof (prove)
goal (1 subgoal):
1.  $\bigwedge t1\ x2\ t2.
  [in\_range\ t1\ u\ v = [x\leftarrow in\_order\ t1.\ u \leq x \wedge x \leq v];
  in\_range\ t2\ u\ v = [x\leftarrow in\_order\ t2.\ u \leq x \wedge x \leq v];\ bst\ t1;\ bst\ t2;\ \forall x \in set\_tree\ t1.\ x < x2;
  \forall x \in set\_tree\ t2.\ x2 < x]
  \implies (u = x2 \wedge x2 = v \longrightarrow (\forall x \in set\_tree\ t1.\ v \leq x \longrightarrow x \neq v) \wedge (\forall x \in set\_tree\ t2.\ v \leq x \longrightarrow x \neq v)) \wedge
  ((u = x2 \longrightarrow x2 \neq v) \longrightarrow
  (u = x2 \wedge x2 < v \longrightarrow$ 
```

Isabelle2017 - tut04.thy

```

41 text <Show that your list is actually in-order>
42 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
43   apply (induction t)
44   apply simp
45   apply (clarsimp simp: filter_empty_conv)

```

```

(x2 < v  $\longrightarrow$ 
x2 # in_range t2 x2 v =
[x # in_order t1 . x2 <= x ^ x <= v] @ x2 # [x # in_order t2 . x2 <= x ^ x <= v]) ^
( $\neg$  x2 < v  $\longrightarrow$ 
( $\forall x \in set\_tree\ t1.\ x2 \leq x \longrightarrow \neg x \leq v$ ) ^ ( $\forall x \in set\_tree\ t2.\ x2 \leq x \longrightarrow \neg x \leq v$ )) ^
((u = x2  $\longrightarrow$   $\neg$  x2 <= v)  $\longrightarrow$ 
(x2 < v  $\longrightarrow$ 
 $\neg$  u < x2  $\longrightarrow$ 
(u = x2 ^ x2 <= v  $\longrightarrow$ 
x2 # in_range t2 x2 v =
[x # in_order t1 . x2 <= x ^ x <= v] @ x2 # [x # in_order t2 . x2 <= x ^ x <= v]) ^

```

Isabelle2017 - tut04.thy

```

41 text <Show that your list is actually in-order>
42 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
43   apply (induction t)
44   apply simp
45   apply (clarsimp simp: filter_empty_conv)
46   apply safe
47   apply simp_all

```

```

proof (prove)
goal (1 subgoal):
1.  $\bigwedge t1\ x2\ t2.
  [in\_range\ t1\ u\ v = [x\leftarrow in\_order\ t1.\ u \leq x \wedge x \leq v];
  in\_range\ t2\ u\ v = [x\leftarrow in\_order\ t2.\ u \leq x \wedge x \leq v];\ bst\ t1;\ bst\ t2;\ \forall x \in set\_tree\ t1.\ x < x2;
  \forall x \in set\_tree\ t2.\ x2 < x]
  \implies (u = x2 \wedge x2 = v \longrightarrow (\forall x \in set\_tree\ t1.\ v \leq x \longrightarrow x \neq v) \wedge (\forall x \in set\_tree\ t2.\ v \leq x \longrightarrow x \neq v)) \wedge$ 
```

Isabelle2017 - tut04.thy

```

41 text <Show that your list is actually in-order>
42 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
43   apply (induction t)
44   apply simp
45   apply (clarsimp simp: filter_empty_conv)
46   apply safe
47   apply simp_all

```

```

proof (prove)
goal (10 subgoals):
1.  $\bigwedge t1\ t2.
  [in\_range\ t1\ v\ v = [x\leftarrow in\_order\ t1.\ v \leq x \wedge x \leq v];
  in\_range\ t2\ v\ v = [x\leftarrow in\_order\ t2.\ v \leq x \wedge x \leq v];\ bst\ t1;\ bst\ t2;\ \forall x \in set\_tree\ t1.\ x < v;
  Ball\ (set\_tree\ t2)\ (op < v); u = v; v \in set\_tree\ t1]
  \implies False$ 
```

Isabelle2017 - tut04.thy (modified)

```

40 text <Show that your list is actually in-order>
41 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u < x \wedge x < v$ ) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply (clarsimp simp: filter_empty_conv)
45 apply safe
46 apply simp_all
47
48
49
50

```

proof (prove)  
goal (10 subgoals):  
1.  $\wedge t1\ t2.$   
[in\_range t1 v = [x--inorder t1 . v  $\leq$  x  $\wedge$  x  $\leq$  v];  
in\_range t2 v = [x--inorder t2 . v  $\leq$  x  $\wedge$  x  $\leq$  v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < v$ ;  
Ball (set\_tree t2) (op < v); u = v; v  $\in$  set\_tree t1]  
 $\implies$  False

47.3 (1178/6485) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 527/1182MB 1:00 PM

Isabelle2017 - tut04.thy

```

40 text <Show that your list is actually in-order>
41 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u < x \wedge x < v$ ) (inorder t)"
42 apply (induction t)
43 apply simp
44 apply (clarsimp simp: filter_empty_conv)
45 apply safe
46 apply simp_all
47 apply auto
48

```

proof (prove)  
goal (1 subgoal):  
1.  $\wedge t1\ x2\ t2.$   
[in\_range t1 u v = [x--inorder t1 . u  $\leq$  x  $\wedge$  x  $\leq$  v];  
in\_range t2 u v = [x--inorder t2 . u  $\leq$  x  $\wedge$  x  $\leq$  v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < x2$ ;  
 $\forall x \in \text{set\_tree } t2. x2 < x$ ]  
 $\implies$  (u = x2  $\wedge$  x2 = v  $\implies$  ( $\forall x \in \text{set\_tree } t1. v \leq x \implies x \neq v$ )  $\wedge$  ( $\forall x \in \text{set\_tree } t2. v \leq x \implies x \neq v$ ))  $\wedge$   
(u = x2  $\implies$  x2  $\neq$  v)  $\implies$   
(u = x2  $\wedge$  x2  $\leq$  v  $\implies$   
(x2 < v  $\implies$

44.32 (1134/6495) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 527/1182MB 1:01 PM

Isabelle2017 - tut04.thy (modified)

```

44 apply (clarsimp simp: filter_empty_conv)
45 apply safe
46 apply simp_all
47 apply (auto simp: filter_empty_conv)
48 thm filter_empty_conv
49
50
51
52

```

(filter ?P ?xs = []) = ( $\forall x \in \text{set } ?xs. \neg ?P\ x$ )

48.22 (1236/6542) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 527/1147MB 1:01 PM

Isabelle2017 - tut04.thy

```

44 text <Show that your list is actually in-order>
45 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u < x \wedge x < v$ ) (inorder t)"
46 apply (induction t)
47 apply simp
48 apply (clarsimp simp: filter_empty_conv)
49 apply safe
50 apply simp_all
51 apply (auto simp: filter_empty_conv filter_empty_conv')
52 done

```

proof (prove)  
goal (1 subgoal):  
1.  $\wedge t1\ x2\ t2.$   
[in\_range t1 u v = [x--inorder t1 . u  $\leq$  x  $\wedge$  x  $\leq$  v];  
in\_range t2 u v = [x--inorder t2 . u  $\leq$  x  $\wedge$  x  $\leq$  v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < x2$ ;  
 $\forall x \in \text{set\_tree } t2. x2 < x$ ]  
 $\implies$  (u = x2  $\wedge$  x2 = v  $\implies$  ( $\forall x \in \text{set\_tree } t1. v \leq x \implies x \neq v$ )  $\wedge$  ( $\forall x \in \text{set\_tree } t2. v \leq x \implies x \neq v$ ))  $\wedge$   
(u = x2  $\implies$  x2  $\neq$  v)  $\implies$   
(u = x2  $\wedge$  x2  $\leq$  v  $\implies$   
(x2 < v  $\implies$

48.13 (1224/6654) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 527/1089MB 1:03 PM

Isabelle2017 - tut04.thy

```

44 text <Show that your list is actually in-order>
45 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
46 apply (induction t)
47 apply simp
48 apply (clarsimp simp: filter_empty_conv)
49 apply safe
50 apply simp_all
51 apply (auto simp: filter_empty_conv filter_empty_conv')
52 done

```

```

[in_range t1 u v = [x←inorder t1 . u ≤ x ∧ x ≤ v];
in_range t2 u v = [x←inorder t2 . u ≤ x ∧ x ≤ v]; bst t1; bst t2;  $\forall x \in \text{set\_tree } t1. x < x2$ ;
 $\forall x \in \text{set\_tree } t2. x2 < x$ ]
 $\implies (u = x2 \wedge x2 = v \implies (\forall x \in \text{set\_tree } t1. v \leq x \implies x \neq v) \wedge (\forall x \in \text{set\_tree } t2. v \leq x \implies x \neq v)) \wedge$ 
 $((u = x2 \implies x2 \neq v) \implies$ 
 $(u = x2 \wedge x2 \leq v \implies$ 
 $(x2 < v \implies$ 
 $x2 \# \text{in\_range } t2 \ x2 \ v =$ 
 $[x \leftarrow \text{inorder } t1 . x2 \leq x \wedge x \leq v] @ x2 \# [x \leftarrow \text{inorder } t2 . x2 \leq x \wedge x \leq v]) \wedge$ 
 $(\neg x2 < v \implies$ 

```

48.13 (1.224/6654) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 0.26/0.089MB 1:04 PM

Isabelle2017 - tut04.thy

```

44 text <Show that your list is actually in-order>
45 Lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u < x \wedge x \leq v$ ) (inorder t)"
46 apply (induction t)
47 apply simp
48 apply (clarsimp simp: filter_empty_conv)
49 apply safe
50 apply simp_all
51 apply (auto simp: filter_empty_conv filter_empty_conv')
52 done

```

```

proof (prove)
goal (26 subgoals):
1.  $\bigwedge t1 \ x2 \ t2 \ x. [in\_range \ t1 \ v \ v = [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v];$ 
 $in\_range \ t2 \ v \ v = [x \leftarrow \text{inorder } t2 . v \leq x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < v;$ 
 $\forall x \in \text{set\_tree } t2. v < x; u = v; v \in \text{set\_tree } t1; v \leq v]$ 
 $\implies \text{False}$ 
2.  $\bigwedge t1 \ x2 \ t2 \ x. [in\_range \ t1 \ v \ v = [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v];$ 
 $in\_range \ t2 \ v \ v = [x \leftarrow \text{inorder } t2 . v < x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < v.$ 

```

49.13 (1.267/6654) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 0.59/0.089MB 1:04 PM

Isabelle2017 - tut04.thy

```

46 apply (induction t)
47 apply simp
48 apply (clarsimp simp: filter_empty_conv)
49 apply safe
50 apply simp_all
51 apply (auto simp: filter_empty_conv filter_empty_conv')
52 done

```

```

proof (prove)
goal (10 subgoals):
1.  $\bigwedge t1 \ t2. [in\_range \ t1 \ v \ v = [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v];$ 
 $in\_range \ t2 \ v \ v = [x \leftarrow \text{inorder } t2 . v \leq x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < v;$ 
 $\text{Ball } (\text{set\_tree } t2) \ (op < v); u = v; v \in \text{set\_tree } t1]$ 
 $\implies \text{False}$ 
2.  $\bigwedge t1 \ t2. [in\_range \ t1 \ v \ v = [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v];$ 
 $in\_range \ t2 \ v \ v = [x \leftarrow \text{inorder } t2 . v < x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < v.$ 

```

50.16 (1.283/6654) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 0.26/0.089MB 1:05 PM

Isabelle2017 - tut04.thy

```

46 apply (induction t)
47 apply simp
48 apply (clarsimp)
49 apply safe
50 apply simp_all
51 apply (auto simp: filter_empty_conv filter_empty_conv')
52 done

```

```

proof (prove)
goal (18 subgoals):
1.  $\bigwedge t1 \ t2. [in\_range \ t1 \ v \ v = [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v];$ 
 $in\_range \ t2 \ v \ v = [x \leftarrow \text{inorder } t2 . v \leq x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < v;$ 
 $\text{Ball } (\text{set\_tree } t2) \ (op < v); u = v]$ 
 $\implies [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v] = []$ 
2.  $\bigwedge t1 \ t2. [in\_range \ t1 \ v \ v = [x \leftarrow \text{inorder } t1 . v \leq x \wedge x \leq v];$ 
 $in\_range \ t2 \ v \ v = [x \leftarrow \text{inorder } t2 . v < x \wedge x \leq v]; \text{bst } t1; \text{bst } t2; \forall x \in \text{set\_tree } t1. x < v.$ 

```

50.17 (1.260/6630) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 0.60/0.089MB 1:05 PM

Isabelle2017 - tut04.thy

```

46 apply (induction t)
47 apply simp
48 apply (clarsimp)
49 apply safe
50 apply simp_all
51 apply (auto simp: filter_empty_conv filter_empty_conv')
52 done

```

in\_range t2 v v = [x←inorder t2 . v ≤ x ∧ x ≤ v]; bst t1; bst t2; ∀xset\_tree t1. x < v;  
 Ball (set\_tree t2) (op < v); u = v] ⇒ [x←inorder t1 . v ≤ x ∧ x ≤ v] = []

2.  $\wedge t1\ t2.$   
 [in\_range t1 v v = [x←inorder t1 . v ≤ x ∧ x ≤ v];  
 in\_range t2 v v = [x←inorder t2 . v ≤ x ∧ x ≤ v]; bst t1; bst t2; ∀xset\_tree t1. x < v;  
 Ball (set\_tree t2) (op < v); u = v] ⇒ [x←inorder t2 . v ≤ x ∧ x ≤ v] = []

3.  $\wedge t1\ x2\ t2.$   
 [in\_range t1 u v = [x←inorder t1 . u < x ∧ x < v];

50.17 (1.260/6630) (isabelle,isabelle,UTF-8-Isabelle)lmmro UG 778/1.00MB 1:05 PM

Isabelle2017 - tut04.thy (modified)

```

48 apply (clarsimp)
49 apply safe
50 apply simp_all
51 apply (simp_all add: filter_empty_conv filter_empty_conv')
52 apply
53 done

```

proof (prove)  
 goal (18 subgoals):  
 1.  $\wedge t1\ t2.$   
 [in\_range t1 v v = [x←inorder t1 . v ≤ x ∧ x ≤ v];  
 in\_range t2 v v = [x←inorder t2 . v ≤ x ∧ x ≤ v]; bst t1; bst t2; ∀xset\_tree t1. x < v;  
 ∀xset\_tree t2. v < x; u = v] ⇒ ∀xset\_tree t1. v ≤ x → x ≠ v

2.  $\wedge t1\ t2.$   
 [in\_range t1 v v = [x←inorder t1 . v ≤ x ∧ x ≤ v];  
 in\_range t2 v v = [x←inorder t2 . v < x ∧ x < v]; bst t1; bst t2; ∀xset\_tree t1. x < v;

52.9 (1.330/6642) Input/output complete (isabelle,isabelle,UTF-8-Isabelle)lmmro UG 534/1.066MB 1:06 PM

Isabelle2017 - tut04.thy (modified)

```

52 apply blast
53 apply blast
54 apply fastforce
55 apply fastforce
56 done

```

text <\Exercise{Pretty Printing of Binary Trees}>

proof (prove)  
 goal:  
 No subgoals!

55.19 (1.386/6698) (isabelle,isabelle,UTF-8-Isabelle)lmmro UG 28/1.019MB 1:07 PM

Isabelle2017 - tut04.thy

```

45 lemma "bst t ⇒ in_range t u v = filter (λx. u < x ∧ x < v) (inorder t)"
46 apply (induction t)
47 apply simp
48 apply (clarsimp)
49 apply safe
50 apply (auto simp: filter_empty_conv filter_empty_conv')
51 done

```

text <\Exercise{Pretty Printing of Binary Trees}>

text <Define a function that checks whether two binary trees have the same structure. The values at the nodes may differ.>

theorem bst ?t ⇒ in\_range ?t ?u ?v = [x←inorder ?t . ?u < x ∧ x < ?v]

52.1 (1.309/6613) (isabelle,isabelle,UTF-8-Isabelle)lmmro UG 608/766MB 1:10 PM



```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
43
44 text <Show that your list is actually in-order>
45 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
46 apply (induction t)
47 apply simp
48 apply (clarsimp)
49 apply safe
50 apply (auto simp: filter_empty_conv filter_empty_conv')
51 done
52
53
54
55 text <\Exercise{Pretty Printing of Binary Trees}>
56

proof (prove)
goal (1 subgoal):
1. bst t  $\implies$  in_range t u v = [x--inorder t . u  $\leq$  x  $\wedge$  x  $\leq$  v]

```

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
42
43 text <Show that your list is actually in-order>
44 lemma "bst t  $\implies$  in_range t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
45 apply (induction t)
46 apply simp
47 apply (clarsimp)
48 apply safe
49 apply (auto simp: filter_empty_conv filter_empty_conv')
50 done
51
52
53
54
55 text <\Exercise{Pretty Printing of Binary Trees}>

proof (prove)
goal (1 subgoal):
1. bst t  $\implies$  in_range t u v = [x--inorder t . u  $\leq$  x  $\wedge$  x  $\leq$  v]

```

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
64 lemma "bst t  $\implies$  set (in_range' t u v) = {x $\in$ set_tree t. u <= x <= v}"
65 apply (induction t)
66 apply auto
67 done
68
69 lemma "bst t  $\implies$  in_range' t u v = filter ( $\lambda x. u \leq x \wedge x \leq v$ ) (inorder t)"
70 apply (induction t)
71 apply (auto simp: filter_empty_conv)
72 done
73
74
75 text <\Exercise{Pretty Printing of Binary Trees}>
76

proof (prove)
goal:
No subgoals!

```

```

Isabelle2017 - tut04.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut04.thy (~/lehre/FDS/SS18/public/exercises/)
52
53
54 fun in_range' :: "'a::linorder tree  $\Rightarrow$  'a  $\Rightarrow$  'a  $\Rightarrow$  'a list"
55 where
56   "in_range' Leaf u v = []"
57   | "in_range' (Node l a r) u v =
58     (if u < a then in_range' r u v
59      else if a > v then in_range' l u v
60      else in_range' l u v @ # in_range' r u v
61     )
62   "
63
64 lemma "bst t  $\implies$  set (in_range' t u v) = {x $\in$ set_tree t. u <= x <= v}"

consts
in_range' :: "'a tree  $\Rightarrow$  'a  $\Rightarrow$  'a  $\Rightarrow$  'a list"
Found termination order: "( $\lambda p. \text{size (fst p)}$ ) < *mlex* > {}"

```

Isabelle2017 - tut04.thy (modified)

```

55 where
56   "in_range' Leaf u v = []"
57 | "in_range' (Node l a r) u v =
58   (if a < u then in_range' r u v
59   else if a > v then in_range' l u v
60   else if a = u then a # in_range' r u v
61   else if a = v then a # in_range' l u v
62   else if a = u & a = v then a # in_range' l u v
63   else if a = u & a = v then a # in_range' r u v
64   else if a = u & a = v then a # in_range' l u v
65   else if a = u & a = v then a # in_range' r u v
66   else if a = u & a = v then a # in_range' l u v
67   else if a = u & a = v then a # in_range' r u v"
68
69 lemma "bst t  $\implies$  set (in_range' t u v) = {x  $\in$  set tree t. u < x  $\wedge$  x < v}"

```

Inner syntax errors  
Failed to parse prop

60.16 (1528/7156) Input/output complete (isabelle.isabelle.UTF-8-isabelle) timr o UG 442/000MB 1:15 PM

Isabelle2017 - tut04.thy (modified)

```

55 where
56   "in_range' Leaf u v = []"
57 | "in_range' (Node l a r) u v =
58   (if a < u then in_range' r u v
59   else if a > v then in_range' l u v
60   else if a = u then a
61   else if a = v then a
62   else if a = u & a = v then a # in_range' r u v
63   else if a = u & a = v then a # in_range' l u v
64   else if a = u & a = v then a # in_range' l u v
65   else if a = u & a = v then a # in_range' r u v
66   else if a = u & a = v then a # in_range' l u v
67   else if a = u & a = v then a # in_range' r u v"
68
69 lemma "bst t  $\implies$  set (in_range' t u v) = {x  $\in$  set tree t. u < x  $\wedge$  x < v}"

```

Inner syntax errors  
Failed to parse prop

60.26 (1538/7166) Input/output complete (isabelle.isabelle.UTF-8-isabelle) timr o UG 508/0300MB 1:15 PM

Isabelle2017 - tut04.thy (modified)

```

55 where
56   "in_range' Leaf u v = []"
57 | "in_range' (Node l a r) u v =
58   (if a < u then in_range' r u v
59   else if a > v then in_range' l u v
60   else if a = u then a # in_range' r u v
61   else if a = v then a # in_range' l u v
62   else if a = u & a = v then a # in_range' l u v
63   else if a = u & a = v then a # in_range' r u v
64   else if a = u & a = v then a # in_range' l u v
65   else if a = u & a = v then a # in_range' r u v
66   else if a = u & a = v then a # in_range' l u v
67   else if a = u & a = v then a # in_range' r u v"
68
69 lemma "bst t  $\implies$  set (in_range' t u v) = {x  $\in$  set tree t. u < x  $\wedge$  x < v}"

```

Inner syntax errors  
Failed to parse prop

61.8 (1564/7192) Input/output complete (isabelle.isabelle.UTF-8-isabelle) timr o UG 565/0300MB 1:16 PM

Isabelle2017 - tut04.thy (modified)

```

55 where
56   "in_range' Leaf u v = []"
57 | "in_range' (Node l a r) u v =
58   (if a < u then in_range' r u v
59   else if a > v then in_range' l u v
60   else if a = u then a # in_range' r u v
61   else if a = v then a # in_range' l u v
62   else if a = u & a = v then a # in_range' l u v @ [a]
63   else if a = u & a = v then a # in_range' r u v
64   else if a = u & a = v then a # in_range' l u v
65   else if a = u & a = v then a # in_range' r u v
66   else if a = u & a = v then a # in_range' l u v
67   else if a = u & a = v then a # in_range' r u v"
68
69 lemma "bst t  $\implies$  set (in_range' t u v) = {x  $\in$  set tree t. u < x  $\wedge$  x < v}"

```

consts  
in\_range' :: "'a tree  $\implies$  'a  $\implies$  'a list"  
Found termination order: "( $\lambda$ p. size (fst p)) <+mlex\*+ {}"

62.13 (1613/7238) Input/output complete (isabelle.isabelle.UTF-8-isabelle) timr o UG 64/877MB 1:17 PM

```

65 where
66   "in_range' Leaf u v = []"
67   | "in_range' (Node l a r) u v =
68     (if a < u then in_range' r u v
69     else if a > v then in_range' l u v
70     else if a = u then a # in_range' r u v
71     else if a = v then in_range' l u v @[a]
72     else.)
73
74   in_range' l u v @ a # in_range' r u v
75 )
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```

consts  
in\_range' :: "'a tree => 'a => 'a list"  
Found termination order: "(λp. size (fst p)) <+mlex\*+ {}"

```

64
65
66 Lemma "bst t => set (in_range' t u v) = {x ∈ set_tree t. u ≤ x ∧ x ≤ v}"
67   apply (induction t)
68   apply auto
69   done
70
71 Lemma "bst t => in_range' t u v = filter (λx. u ≤ x ∧ x ≤ v) (inorder t)"
72   apply (induction t)
73   apply (auto simp: filter_empty_conv filter_empty_conv')

```

theorem bst ?t => set (in\_range' ?t ?u ?v) = {x ∈ set\_tree ?t. ?u ≤ x ∧ x ≤ ?v}

```

74 done
75
76
77 text <\Exercise{Pretty Printing of Binary Trees}>
78
79 text <Define a function that checks whether two binary trees have the same
80 structure. The values at the nodes may differ.>
81 fun bin_tree2 :: "'a tree => 'b tree => bool"
82 where
83   "bin_tree2 Leaf Leaf <=> True"
84   | "bin_tree2 ()"
85
86 text <While this function itself is not very useful, the induction rule generated by the function package
87 It allows simultaneous induction over two trees:>
88 print_statement bin_tree2.induct
89
90 text <Binary trees can be uniquely pretty-printed by emitting a symbol L
91 for a leaf, and a symbol N for a node. Each N is followed by
92 the pretty-prints of the left and right tree. No additional brackets
93 are required!>
94 datatype 'a tchar = L | N 'a

```

```

74 done
75
76
77 text <\Exercise{Pretty Printing of Binary Trees}>
78
79 text <Define a function that checks whether two binary trees have the same
80 structure. The values at the nodes may differ.>
81 fun bin_tree2 :: "'a tree => 'b tree => bool"
82 where
83   "bin_tree2 Leaf Leaf <=> True"
84   | "bin_tree2 (Node l1 _ r1) (Node l2 _ r2) <=> bin_tree2 l1 l2 ∧ bin_tree2 r1 r2"
85   | "bin_tree2 _ _ <=> False"
86
87
88
89 text <While this function itself is not very useful, the induction rule generated by the function package
90 It allows simultaneous induction over two trees:>
91 print_statement bin_tree2.induct
92
93 text <Binary trees can be uniquely pretty-printed by emitting a symbol L
94 for a leaf, and a symbol N for a node. Each N is followed by

```

Isabelle2017 - tut04.thy

```

| "bin_tree2 _ _ <math>\rightarrow</math> False"

text <While this function itself is not very useful, the induction rule generated by the function package allows simultaneous induction over two trees:>
print_statement bin_tree2.induct

text <Binary trees can be uniquely pretty-printed by emitting a symbol L

```

```

theorem induct:
  fixes P :: "'a tree => 'b tree => bool"
  and a0 :: "'a tree"
  and a1 :: "'b tree"
  assumes "P () ()"
  and "\l1 uu_ r1 l2 uv_ r2. [P l1 l2; P r1 r2] => P (l1, uu_, r1) (l2, uv_, r2)"
  and "\v va vb. P (v, va, vb) ()"
  and "\v va vb. P () (v, va, vb)"
  shows "P a0 a1"

```

91.35 (2525/7354) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 00/801MB 1:22 PM

Isabelle2017 - tut04.thy

```

print_statement bin_tree2.induct

text <Binary trees can be uniquely pretty-printed by emitting a symbol L
for a leaf, and a symbol N for a node. Each N is followed by
the pretty-prints of the left and right tree. No additional brackets
are required!>

datatype 'a tchar = L | N 'a

fun pretty :: "'a tree => 'a tchar list"
  where
  "pretty _ = undefined"

```

98.31 (2790/7354) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 06/701MB 1:23 PM

Isabelle2017 - tut04.thy

```

where
  "pretty _ = undefined"

text <Show that pretty-printing is actually unique, i.e.,
no two different trees are pretty-printed the same way.
Hint: Auxiliary lemma. Simultaneous induction over both trees.
>

lemma pretty_unique: "pretty t = pretty t' => t=t'"
oops

text <\Exercise[Enumeration of Trees]>

```

109.39 (3102/7354) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 33/787MB 1:24 PM

Isabelle2017 - tut04.thy

```

datatype 'a tchar = L | N 'a

fun pretty :: "'a tree => 'a tchar list"
  where
  "pretty Leaf = [L]"
  | "pretty (Node l a r) = N a # pretty l @ pretty r"

text <Show that pretty-printing is actually unique, i.e.,
no two different trees are pretty-printed the same way.
Hint: Auxiliary lemma. Simultaneous induction over both trees.
>

consts
  pretty :: "'a tree => 'a tchar list"
  Found termination order: "size <math>\langle</math>mlex</math>> {}"

```

104.1 (2926/7423) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 05/776MB 1:26 PM

```

Isabelle2017 - tut04.thy
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tut04.thy (~/lehre/FDS/SS18/public/exercises/)
97 datatype 'a tchar = L | N 'a
98
99
100 fun pretty :: "'a tree => 'a tchar list"
101 where
102   "pretty Leaf = [L]"
103 | "pretty (Node l a r) = N a # pretty l @ pretty r"
104
105
106 text <Show that pretty-printing is actually unique, i.e.,
107 no two different trees are pretty-printed the same way.
108 Hint: Auxiliary lemma. Simultaneous induction over both trees.
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Isabelle2017 - tut04.thy

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109 Hint: Auxiliary lemma. Simultaneous induction over both trees.
110 >
111
112 lemma pretty_unique: "pretty t = pretty t'  $\implies$  t=t'"
113   apply (induction t t' rule: bin_tree2.induct)
114   apply auto
115   sledgehammer
116
117 text <\Exercise{Enumeration of Trees}>

```

Proof state:  Auto update:  Update: Search: 100%

```

proof (prove)
goal (2 subgoals):
1.  $\bigwedge l1\ r1\ l2\ r2.$ 
   [pretty l1 = pretty l2  $\implies$  l1 = l2; pretty r1 = pretty r2  $\implies$  r1 = r2;
   pretty l1 @ pretty r1 = pretty l2 @ pretty r2]
 $\implies$  l1 = l2
2.  $\bigwedge l1\ r1\ l2\ r2.$ 
   [pretty l1 = pretty l2  $\implies$  l1 = l2; pretty r1 = pretty r2  $\implies$  r1 = r2;
   pretty l1 @ pretty r1 = pretty l2 @ pretty r2]

```

114.15 (3238/7484) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 249/753MB 1:32 PM

Isabelle2017 - tut04.thy

```

109 Hint: Auxiliary lemma. Simultaneous induction over both trees.
110 >
111
112 lemma pretty_unique: "pretty t = pretty t'  $\implies$  t=t'"
113   apply (induction t t' rule: bin_tree2.induct)
114   apply auto
115
116
117 text <\Exercise{Enumeration of Trees}>

```

Proof state:  Auto update:  Update: Search: 100%

```

proof (prove)
goal (1 subgoal):
1. pretty t = pretty t'  $\implies$  t = t'

```

112.33 (3153/7468) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 322/744MB 1:33 PM

Isabelle2017 - tut04.thy

```

109 Hint: Auxiliary lemma. Simultaneous induction over both trees.
110 >
111
112 lemma pretty_unique: "pretty t @ xs = pretty t' @ xs'  $\implies$  t=t'"
113   apply (induction t t' rule: bin_tree2.induct)
114   apply auto
115
116
117 text <\Exercise{Enumeration of Trees}>

```

Proof state:  Auto update:  Update: Search: 100%

```

proof (prove)
goal (1 subgoal):
1. pretty t @ xs = pretty t' @ xs'  $\implies$  t = t'

```

112.31 (3151/7480) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 40/734MB 1:34 PM

Isabelle2017 - tut04.thy

```

109 Hint: Auxiliary lemma. Simultaneous induction over both trees.
110 >
111
112 lemma pretty_unique: "pretty t @ xs = pretty t' @ xs'  $\implies$  t=t'"
113   apply (induction t t' rule: bin_tree2.induct)
114   apply auto
115
116
117 text <\Exercise{Enumeration of Trees}>

```

Proof state:  Auto update:  Update: Search: 100%

```

proof (prove)
goal (2 subgoals):
1.  $\bigwedge l1\ r1\ l2\ r2.$ 
   [pretty l1 @ xs = pretty l2 @ xs'  $\implies$  l1 = l2;
   pretty r1 @ xs = pretty r2 @ xs'  $\implies$  r1 = r2;
   pretty l1 @ pretty r1 @ xs = pretty l2 @ pretty r2 @ xs']
 $\implies$  l1 = l2
2.  $\bigwedge l1\ r1\ l2\ r2.$ 
   [pretty l1 @ xs = pretty l2 @ xs'  $\implies$  l1 = l2;
   pretty r1 @ xs = pretty r2 @ xs'  $\implies$  r1 = r2;
   pretty l1 @ pretty r1 @ xs = pretty l2 @ pretty r2 @ xs']

```

114.15 (3250/7480) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 35/734MB 1:35 PM

Isabelle2017 - tut04.thy

```

109 >
110 Hint: Auxiliary lemma. Simultaneous induction over both trees.
111
112 lemma pretty_unique: "pretty t @ xs = pretty t' @ xs'  $\implies$  t=t'"
113 apply (induction t t' rule: bin_tree2.induct)
114 apply auto
115
116
117 text <\Exercise{Enumeration of Trees}>
118
119
120 proof (prove)
121 goal (4 subgoals):
122 1. pretty () @ xs = pretty () @ xs'  $\implies$  () = ()
123 2.  $\bigwedge l1\ u\ r1\ l2\ uv\_ r2.$ 
124   [pretty l1 @ xs = pretty l2 @ xs'  $\implies$  l1 = l2;
125   pretty r1 @ xs = pretty r2 @ xs'  $\implies$  r1 = r2;
126   pretty (l1, uv_, r1) @ xs = pretty (l2, uv_, r2) @ xs']
127  $\implies$  (l1, uv_, r1) = (l2, uv_, r2)

```

113.26 (3211/7480) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 2017/34MB 1:37 PM

Isabelle2017 - tut04.thy (modified)

```

109 >
110 Hint: Auxiliary lemma. Simultaneous induction over both trees.
111
112 lemma pretty_unique: "pretty t @ xs = pretty t' @ xs'  $\implies$  t=t'"
113 apply (induction t t' arbitrary: xs xs' rule: bin_tree2.induct)
114 apply auto
115
116
117
118
119
120 proof (prove)
121 goal (1 subgoal):
122 1.  $\bigwedge l1\ r1\ l2\ r2\ xs\ xs'.$ 
123   [ $\bigwedge xs\ xs'. pretty\ l1\ @\ xs = pretty\ l2\ @\ xs' \implies l1 = l2;$ 
124    $\bigwedge xs\ xs'. pretty\ r1\ @\ xs = pretty\ r2\ @\ xs' \implies r1 = r2;$ 
125   pretty l1 @ pretty r1 @ xs = pretty l2 @ pretty r2 @ xs']
126  $\implies r1 = r2$ 

```

115.5 (3273/7503) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 3627/34MB 1:38 PM

Isabelle2017 - tut04.thy

```

109 >
110 Hint: Auxiliary lemma. Simultaneous induction over both trees.
111
112 lemma pretty_unique: "pretty t @ xs = pretty t' @ xs'  $\implies$  t=t'"
113 apply (induction t t' arbitrary: xs xs' rule: bin_tree2.induct)
114 apply auto
115 sledgehammer
116
117
118
119
120 proof (prove)
121 goal (1 subgoal):
122 1.  $\bigwedge l1\ r1\ l2\ r2\ xs\ xs'.$ 
123   [ $\bigwedge xs\ xs'. pretty\ l1\ @\ xs = pretty\ l2\ @\ xs' \implies l1 = l2;$ 
124    $\bigwedge xs\ xs'. pretty\ r1\ @\ xs = pretty\ r2\ @\ xs' \implies r1 = r2;$ 
125   pretty l1 @ pretty r1 @ xs = pretty l2 @ pretty r2 @ xs']
126  $\implies r1 = r2$ 

```

114.15 (3268/7515) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 1077/34MB 1:38 PM

Isabelle2017 - tut04.thy

```

122 text <Write a function that generates the set of all trees up to a given height.
123 Show that only trees up to the specified height are contained.
124
125 (The other direction, i.e., that all trees are contained, requires an
126 advanced case split, which has not yet been introduced in the lecture,
127 so it is omitted here)
128
129
130 fun enum :: "nat  $\implies$  unit tree set" where
131   "enum _ = undefined"
132
133 lemma enum_sound: "t  $\in$  enum n  $\implies$  height t  $\leq$  n"

```

130.2 (3668/7527) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 3/716MB 1:40 PM

Isabelle2017 - tut04.thy (modified)

```

124 (The other direction, i.e., that all trees are contained, requires an
125 advanced case split, which has not yet been introduced in the lecture,
126 so it is omitted here)
127
128
129
130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {}"
132
133 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
134
135  oops
  
```

```

consts
  enum :: "nat ⇒ unit tree set"
Missing patterns in function definition:
  \v. enum (Suc v) = undefined
Found termination order: "{}"
  
```

131.14 (3718/7520) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 27/71.6MB 1:40 PM

Isabelle2017 - tut04.thy (modified)

```

124 (The other direction, i.e., that all trees are contained, requires an
125 advanced case split, which has not yet been introduced in the lecture,
126 so it is omitted here)
127
128
129
130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   "enum (Suc n) = e"
133
134 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
135
136  oops
  
```

Inner syntax error: unexpected end of input  
Failed to parse prop

132.20 (3744/7545) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 96/706MB 1:41 PM

Isabelle2017 - tut04.thy (modified)

```

127 so it is omitted here)
128
129
130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   "enum (Suc n) = (enum n U {Nod})"
133
134 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
135
136  oops
  
```

```

consts
  enum :: "nat ⇒ unit tree set"
Found termination order: "size <+mlex> {}"
  
```

132.33 (3757/7562) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 115/716MB 1:42 PM

Isabelle2017 - tut04.thy

```

127 so it is omitted here)
128
129
130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   "enum (Suc n) = (enum n U {Nod l () r | l r. l.enum n ∧ r.enum n})"
133
134 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
135
136  oops
  
```

```

consts
  enum :: "nat ⇒ unit tree set"
Found termination order: "size <+mlex> {}"
  
```

132.45 (3769/7598) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 331/399MB 1:43 PM



Isabelle2017 - tut04.thy (modified)

```

127 so it is omitted here)
128 >
129
130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   | "enum (Suc n) = (enum n U {Node l () r | l r. l ∈ enum n ∧ r ∈ enum n})"
133
134
135 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
136
137 oops
  
```

Variable "r" occurs on right hand side only:  
 $\wedge n r. \text{enum} (\text{Suc } n) = \text{enum } n \cup \{(\text{l}, (), \text{r}) \mid \text{l}. \text{l} \in \text{enum } n \wedge \text{r} \in \text{enum } n\}$

132.47 (3771/7598) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 369/99MB 1:43 PM

Isabelle2017 - tut04.thy

```

127 so it is omitted here)
128 >
129
130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   | "enum (Suc n) = (enum n U {Node l () r | l r. l ∈ enum n ∧ r ∈ enum n})"
133
134
135 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
136
137 oops
  
```

consts  
enum :: "nat ⇒ unit tree set"  
Found termination order: "size < \*mlex> {}"

132.48 (3772/7598) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 85/689MB 1:44 PM

Isabelle2017 - tut04.thy (modified)

```

130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   | "enum (Suc n) = (enum n U {Node l () r | l r. l ∈ enum n ∧ r ∈ enum n})"
133
134
135 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
136
137
138
139
140
141
  
```

proof (prove)  
goal (1 subgoal):  
1.  $t \in \text{enum } n \Rightarrow \text{height } t \leq n$

137.3 (3848/7594) Input/output complete (isabelle,isabelle,UTF-8-isabelle)tmr o UG 4/689MB 1:45 PM

Isabelle2017 - tut04.thy (modified)

```

130 fun enum :: "nat ⇒ unit tree set" where
131   "enum 0 = {Leaf}"
132   | "enum (Suc n) = (enum n U {Node l () r | l r. l ∈ enum n ∧ r ∈ enum n})"
133
134
135 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
136
137 apply
  
```

proof (prove)  
goal (1 subgoal):  
1.  $t \in \text{enum } n \Rightarrow \text{height } t \leq n$

137.9 (3854/7600) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 7/689MB 1:45 PM

Isabelle2017 - tut04.thy

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131 "enum 0 = {Leaf}"
132 | "enum (Suc n) = (enum n U {Node l () r | l r. l ∈ enum n ∧ r ∈ enum n })"
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136 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
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proof (prove)  
goal (1 subgoal):  
1.  $\bigwedge n t. [\bigwedge t. t \in \text{enum } n \Rightarrow \text{height } t \leq n; t \in \text{enum } n] \Rightarrow \text{height } t \leq \text{Suc } n$

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132 | "enum (Suc n) = (enum n U {Node l () r | l r. l ∈ enum n ∧ r ∈ enum n })"
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136 lemma enum_sound: "t ∈ enum n ⇒ height t ≤ n"
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138   apply auto
139   by fastforce
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theorem enum\_sound:  $?t \in \text{enum } ?n \Rightarrow \text{height } ?t \leq ?n$

ex04.pdf

Submission until Friday, May 26, 11:59am.

In this homework, we will develop a binary search tree that additionally stores the rank (= number of nodes) of the left subtree in each node.

With this auxiliary information, it is easy to implement a rank query, i.e., to return the position of a given element in the inorder traversal.

**datatype**  $'a \text{ rtree} = \text{Leaf} \mid \text{Node } "'a \text{ rtree} \text{ nat } "'a \text{ rtree}$

Define a function to count the number of nodes in a tree

**fun**  $\text{num\_nodes} :: "'a \text{ rtree} \Rightarrow \text{nat}$  **where**

Define a function to check for the invariant: search tree property and the correct rank annotation (number of nodes in left subtree)

**fun**  $\text{rbst} :: "'a::\text{linorder } \text{rtree} \Rightarrow \text{bool}$  **where**

Define the insert function. You may assume that the value to be inserted is not contained in the tree. Note: Double-check to correctly update the rank annotation.

**fun**  $\text{rins} :: "'a::\text{linorder} \Rightarrow 'a \text{ rtree} \Rightarrow 'a \text{ rtree}$  **where**

Show that  $\text{rins}$  actually inserts, and preserves the invariant. Hint: Auxiliary lemma on number of nodes.

**lemma**  $\text{rins\_set: } \text{set\_rtree } (\text{rins } x \ t) = \text{insert } x \ (\text{set\_rtree } t)$

**lemma**  $x \notin \text{set\_rtree } t \Rightarrow \text{rbst } t \Rightarrow \text{rbst } (\text{rins } x \ t)$

ex04.pdf

**datatype**  $'a \text{ rtree} = \text{Leaf} \mid \text{Node } "'a \text{ rtree} \text{ nat } "'a \text{ rtree}$

Define a function to count the number of nodes in a tree

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Define a function to check for the invariant: search tree property and the correct rank annotation (number of nodes in left subtree)

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**fun**  $\text{rins} :: "'a::\text{linorder} \Rightarrow 'a \text{ rtree} \Rightarrow 'a \text{ rtree}$  **where**

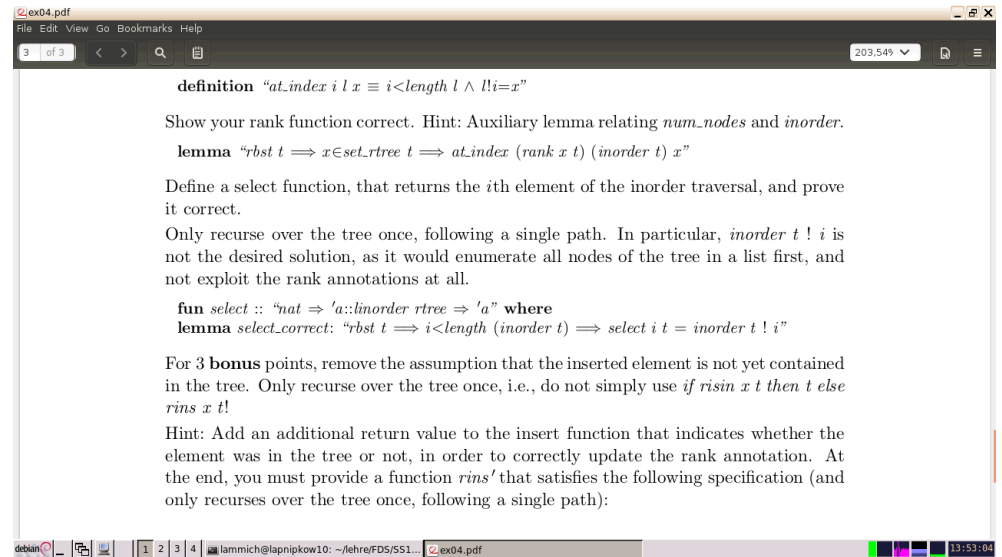
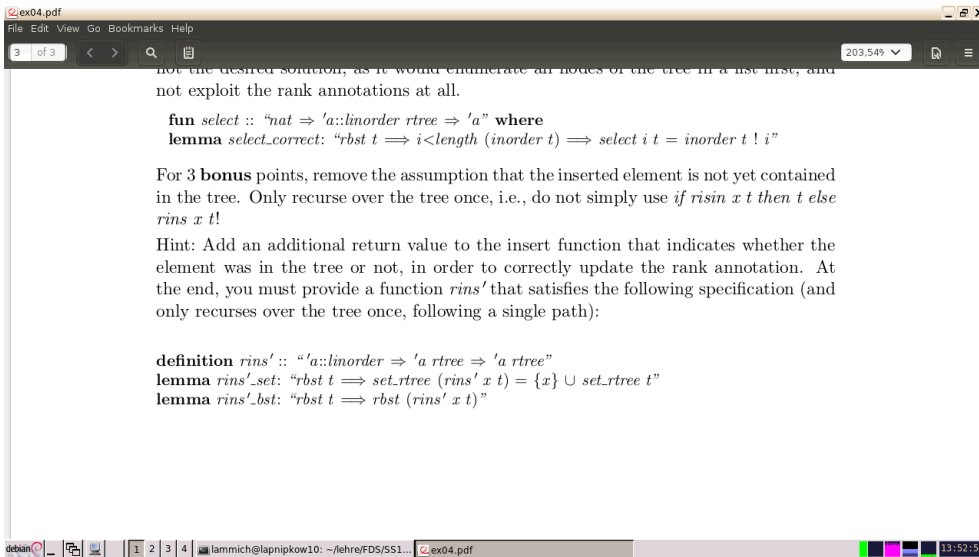
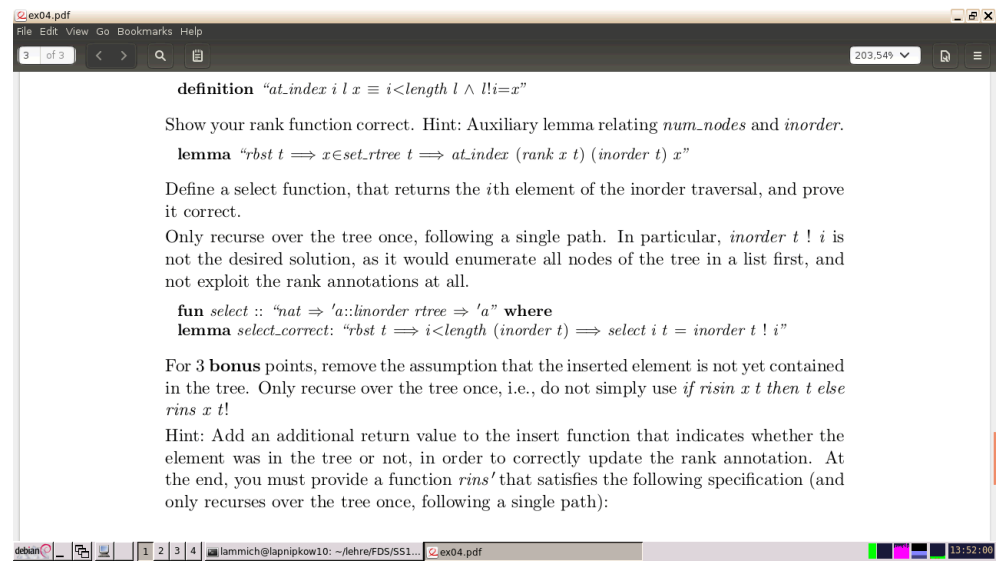
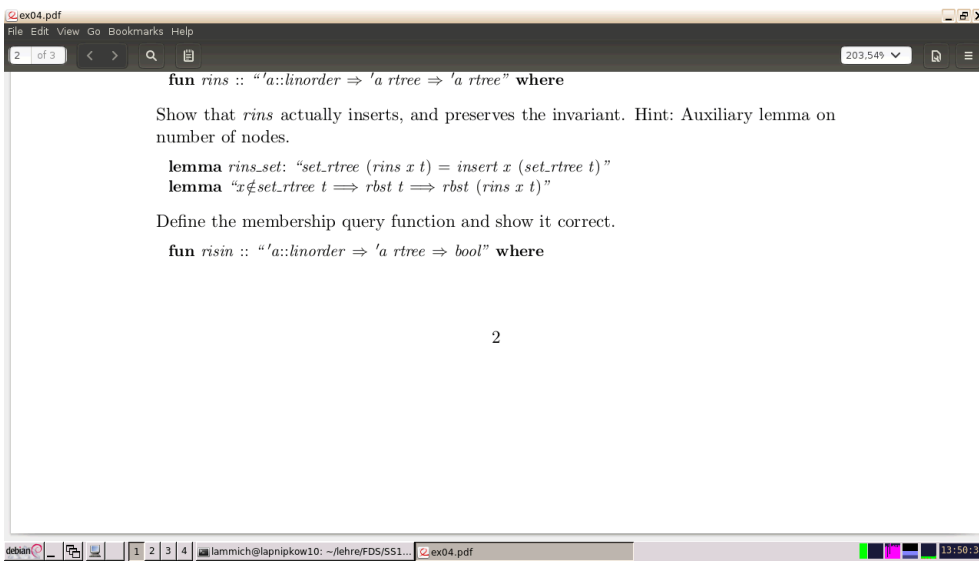
Show that  $\text{rins}$  actually inserts, and preserves the invariant. Hint: Auxiliary lemma on number of nodes.

**lemma**  $\text{rins\_set: } \text{set\_rtree } (\text{rins } x \ t) = \text{insert } x \ (\text{set\_rtree } t)$

**lemma**  $x \notin \text{set\_rtree } t \Rightarrow \text{rbst } t \Rightarrow \text{rbst } (\text{rins } x \ t)$

Define the membership query function and show it correct.

**fun**  $\text{risin} :: "'a::\text{linorder} \Rightarrow 'a \text{ rtree} \Rightarrow \text{bool}$  **where**



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2 of 3 203.54%

Show that pretty-printing is actually unique, i.e., no two different trees are pretty-printed the same way. Hint: Auxiliary lemma. Simultaneous induction over both trees.

**lemma** *pretty\_unique*: “pretty t = pretty t'  $\implies$  t=t'”

**Exercise 4.3** Enumeration of Trees

Write a function that generates the set of all trees up to a given height. Show that only trees up to the specified height are contained.  
(The other direction, i.e., that all trees are contained, requires an advanced case split, which has not yet been introduced in the lecture, so it is omitted here)

**fun** *enum* :: “nat  $\Rightarrow$  unit tree set” **where**  
**lemma** *enum\_sound*: “t  $\in$  enum n  $\implies$  height t  $\leq$  n”

**Homework 4** Rank Annotated Trees

*Submission until Friday, May 26, 11:59am.*

debian 1 2 3 4 tammich@lapnikow10: ~/lehre/FDS/SS1... ex04.pdf 14:04:04