Script generated by TTT

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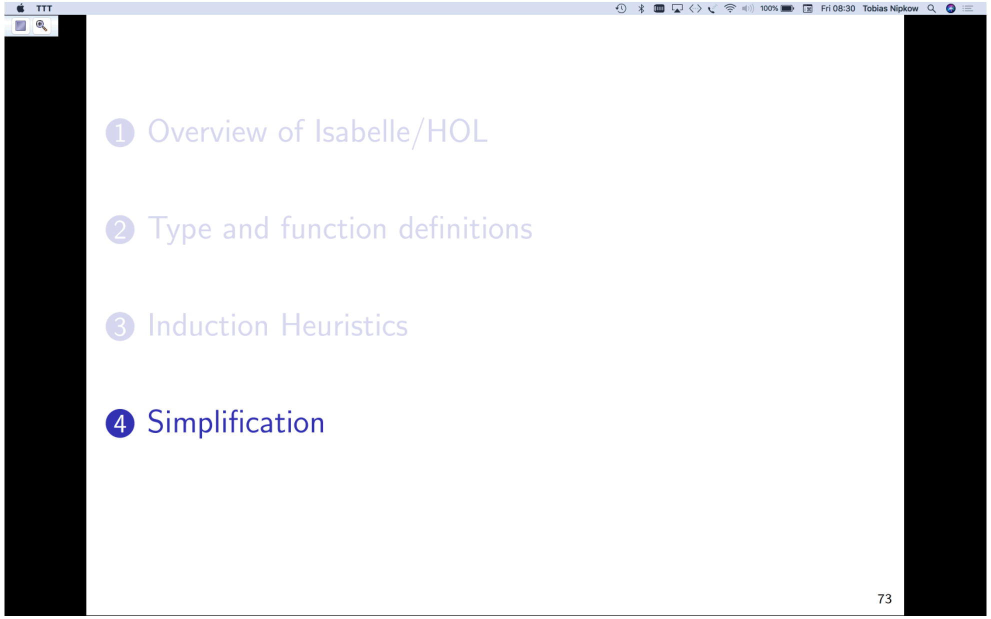
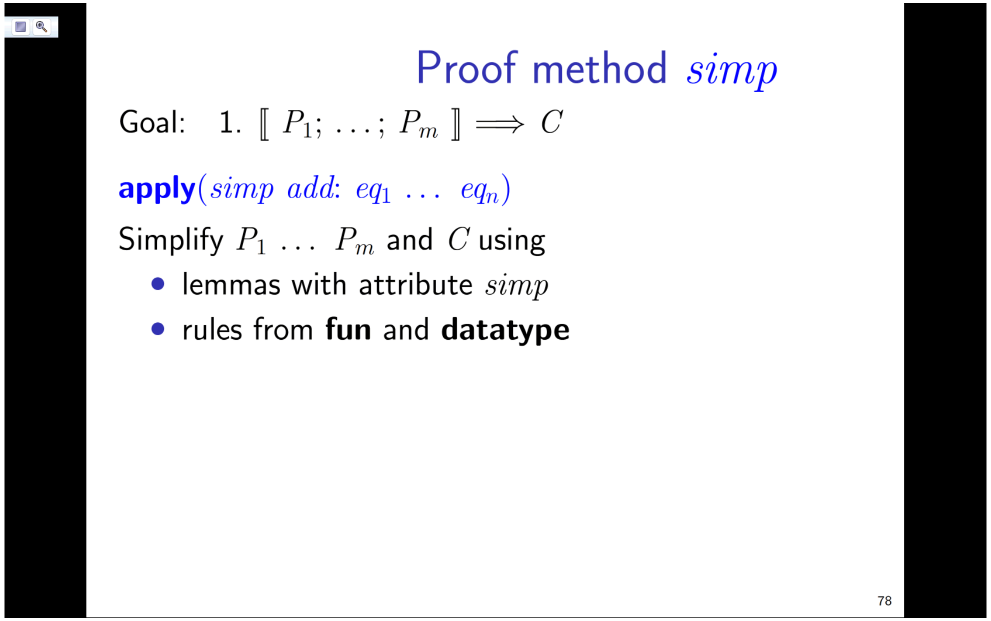


1. Overview of Isabelle/HOL
2. Type and function definitions
3. Induction Heuristics
4. Simplification



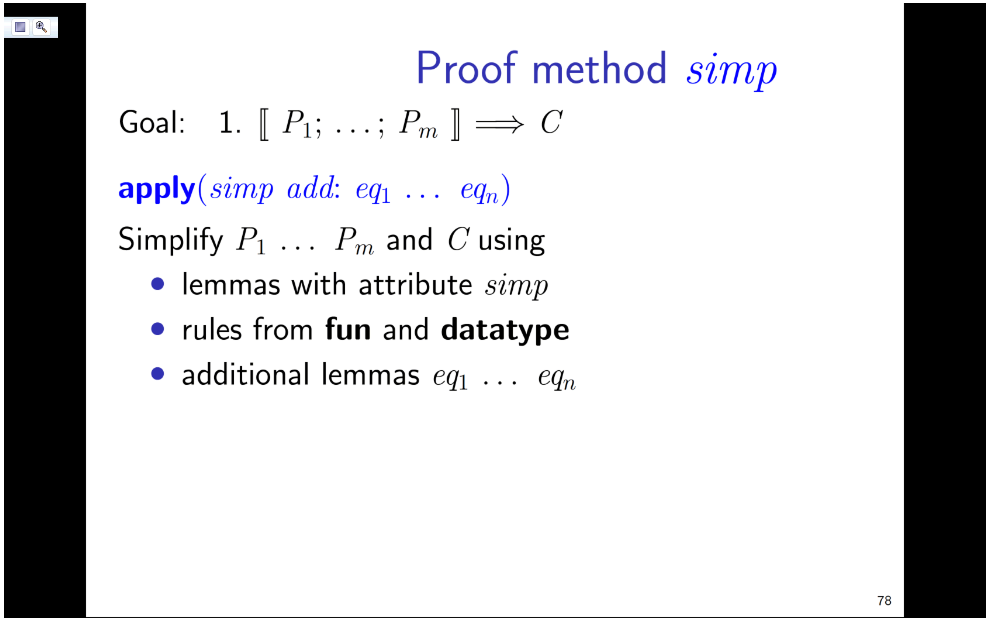
Proof method *simp*

Goal: 1. $[\![P_1; \ldots; P_m]\!] \Longrightarrow C$

apply$(simp\ add\!: eq_1\ \ldots\ eq_n)$

Simplify $P_1\ \ldots\ P_m$ and C using

- lemmas with attribute *simp*
- rules from **fun** and **datatype**



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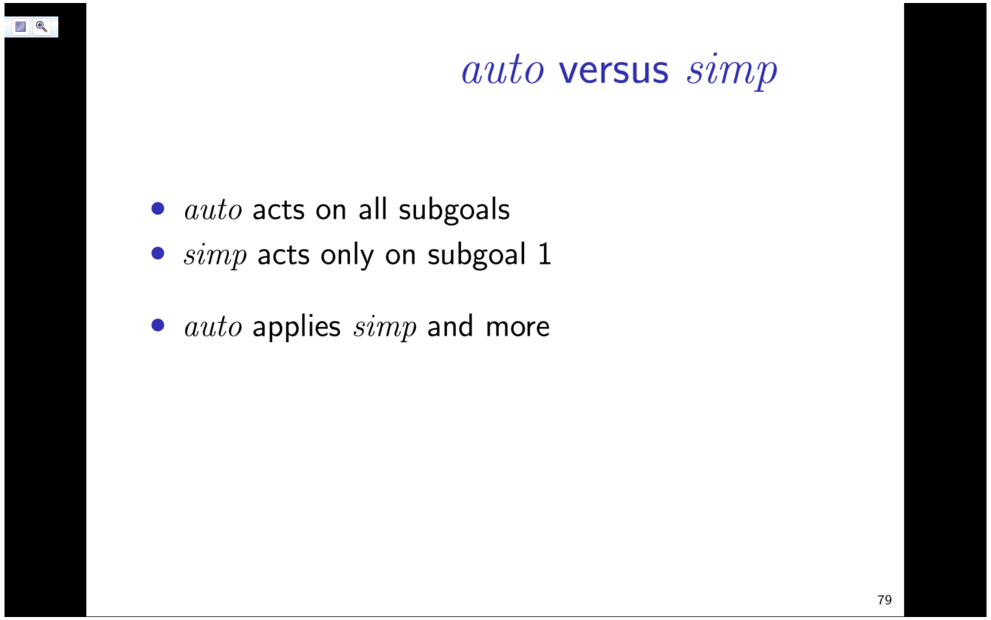
Variations:

- $(simp\ \ldots\ del\colon\ \ldots)$ removes *simp*-lemmas
- *add* and *del* are optional

auto versus *simp*

- *auto* acts on all subgoals
- *simp* acts only on subgoal 1



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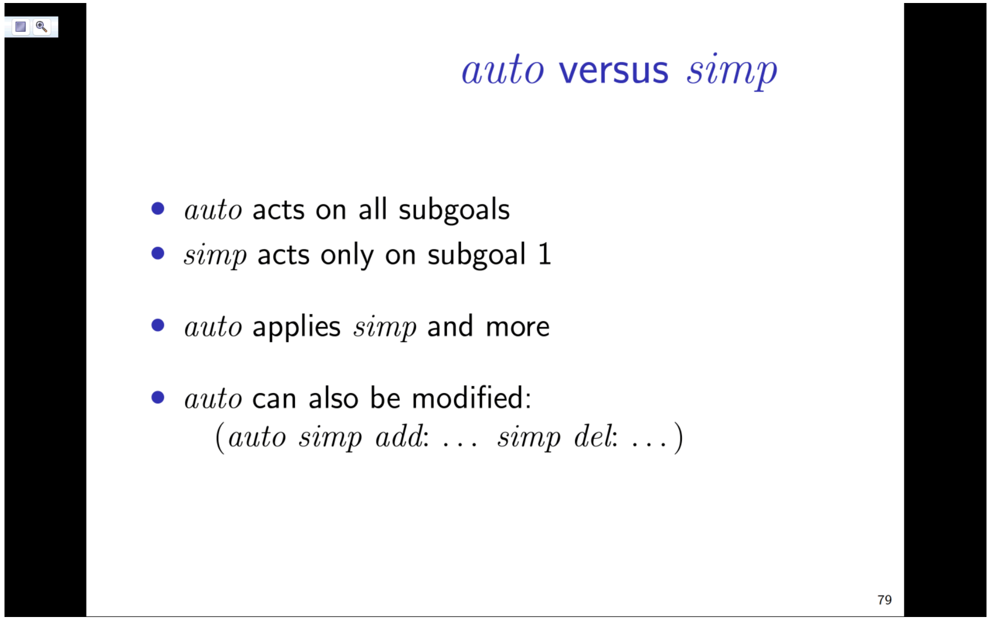

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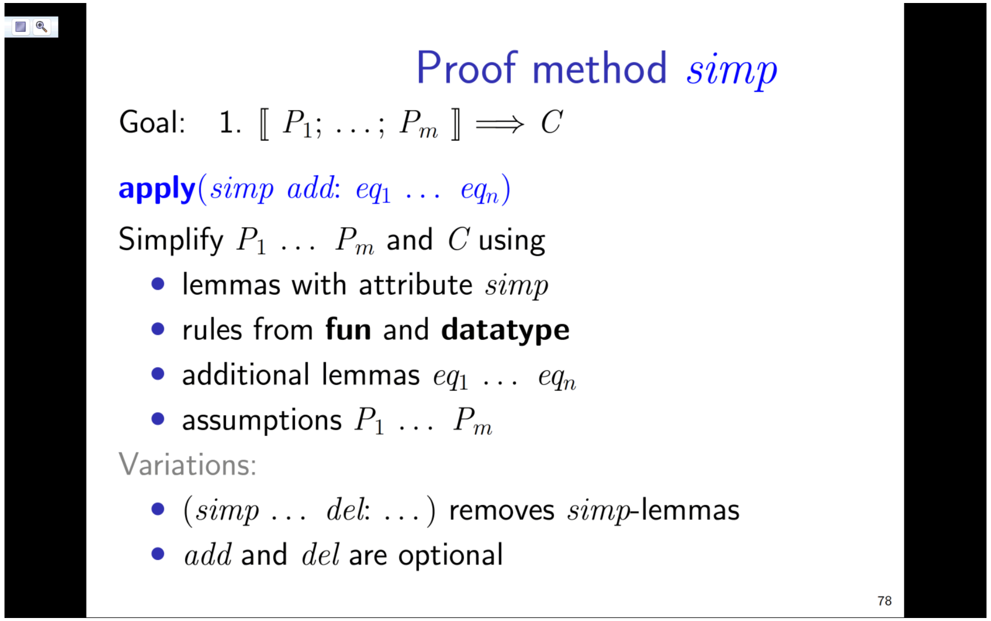

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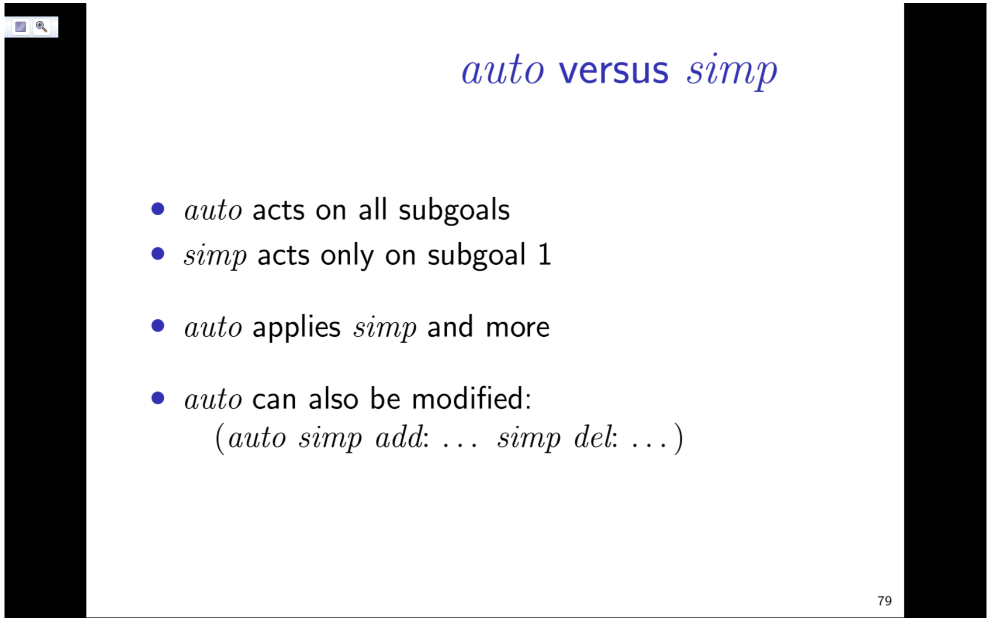

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Rewriting with definitions

Definitions (**definition**) must be used explicitly:

$$(\textit{simp add: f\_def} \dots)$$

Case splitting with *simp/auto*

Automatic:

$$P\ (\textsf{\textit{if}}\ A\ \textsf{\textit{then}}\ s\ \textsf{\textit{else}}\ t) \\ = \\ (A \longrightarrow P(s)) \wedge (\neg A \longrightarrow P(t))$$

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By hand:

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Proof method: *(simp split: nat.split)*
Or *auto*. Similar for any datatype *t*: *t.split*

Splitting pairs with *simp/auto*

How to replace

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Proof method: $(\textit{simp split: prod.split})$

`Simp_Demo.thy`

Preview: sets

Type: $'a\ set$

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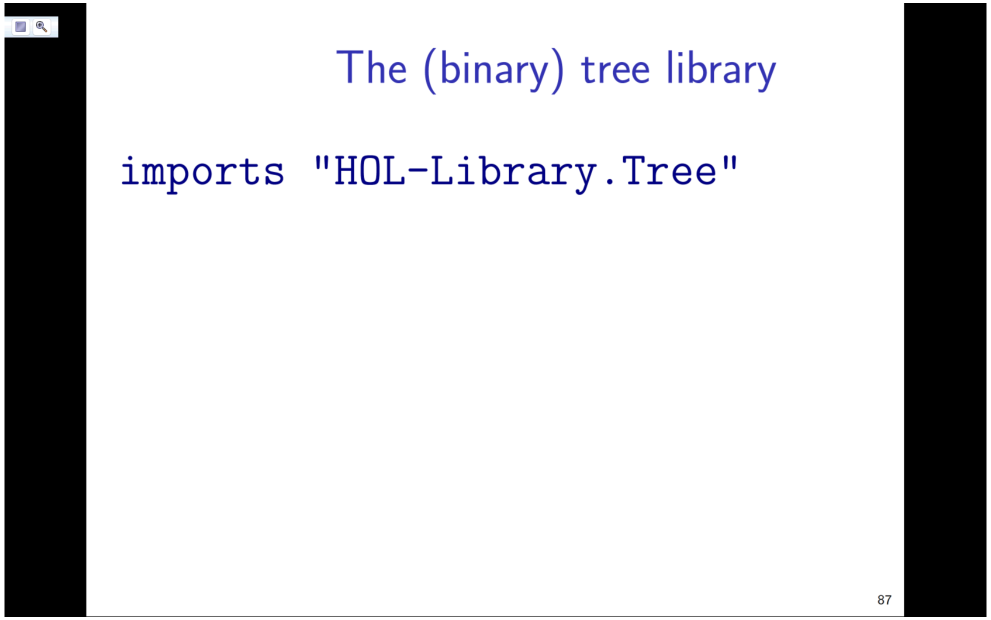
Operations: $a \in A$, $A \cup B$, . . .

Bounded quantification: $\forall\, a{\in}A.\ P$

Proof method $auto$ knows (a little) about sets.



The (binary) tree library

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imports "HOL-Library.Tree"
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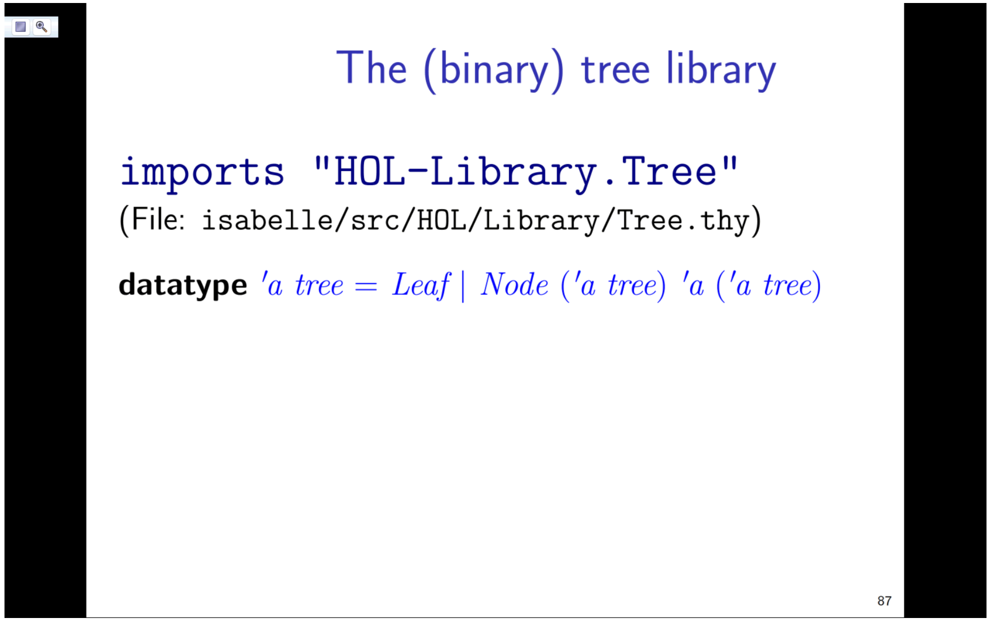
(File: `isabelle/src/HOL/Library/Tree.thy`)



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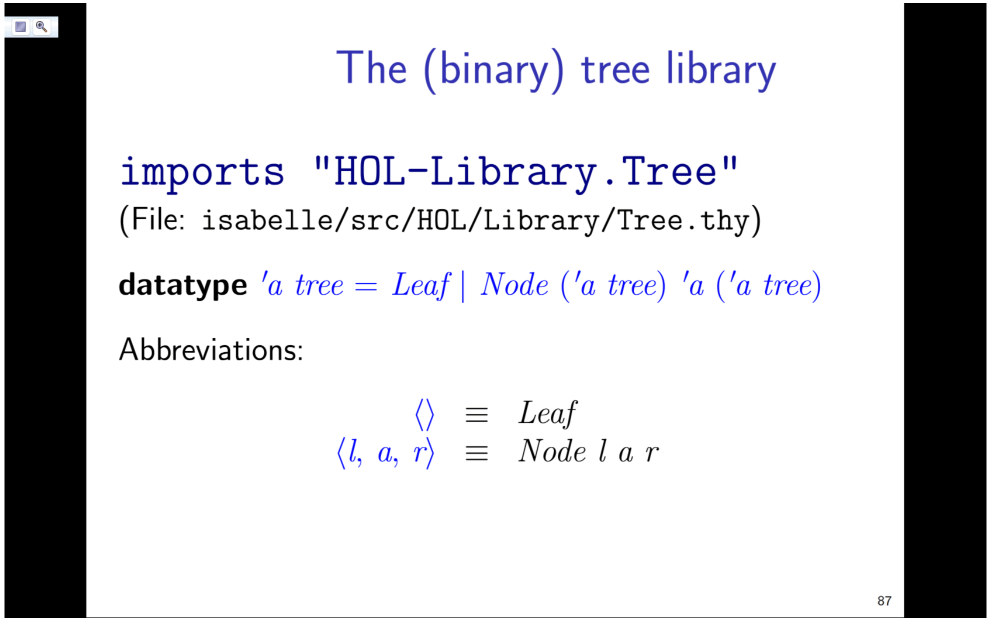
datatype $'a\ tree = Leaf \mid Node\ ('a\ tree)\ 'a\ ('a\ tree)$



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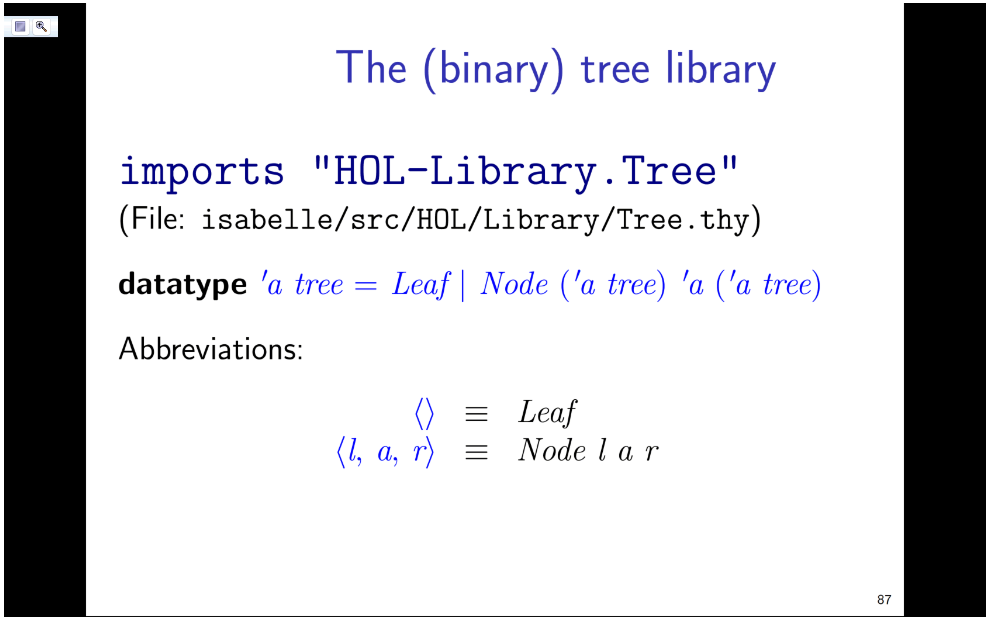
Abbreviations:

$$\begin{array}{rcl} \langle\rangle & \equiv & Leaf \\ \langle l,\ a,\ r\rangle & \equiv & Node\ l\ a\ r \end{array}$$



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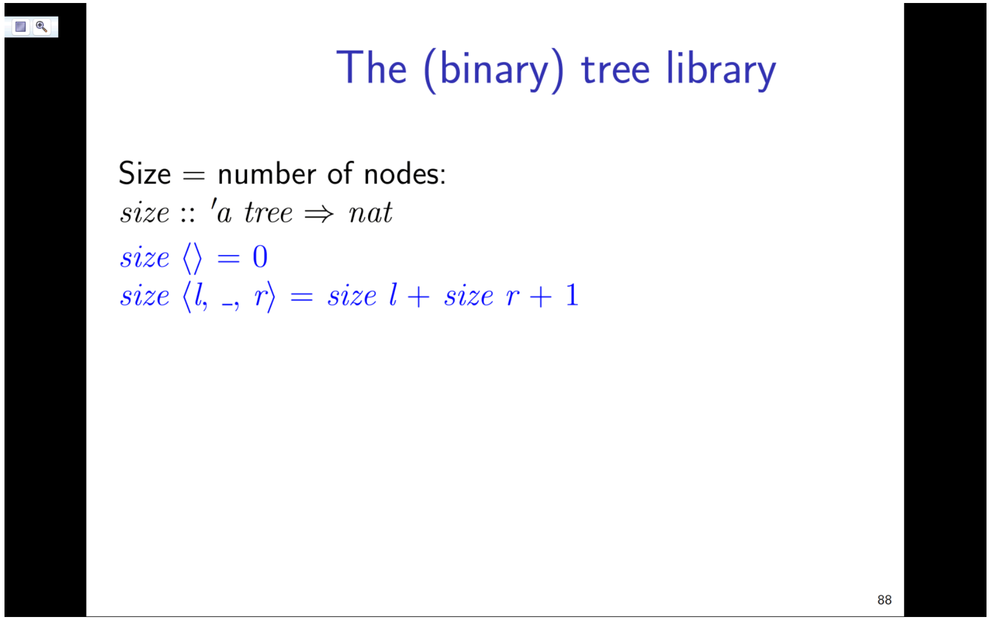


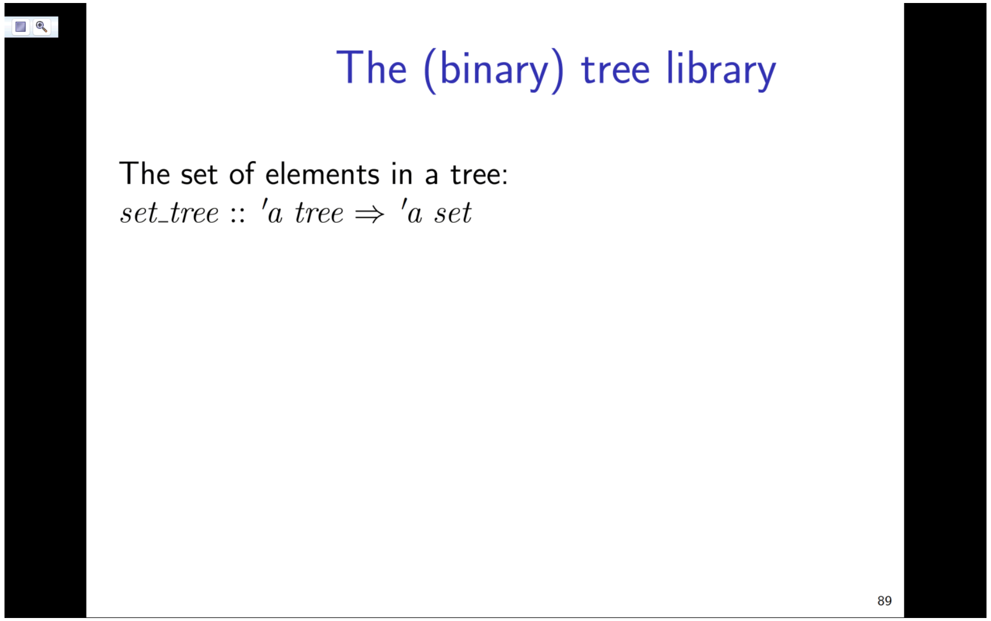
The (binary) tree library

Size = number of nodes:

$size :: 'a\ tree \Rightarrow nat$

$size\ \langle\rangle = 0$

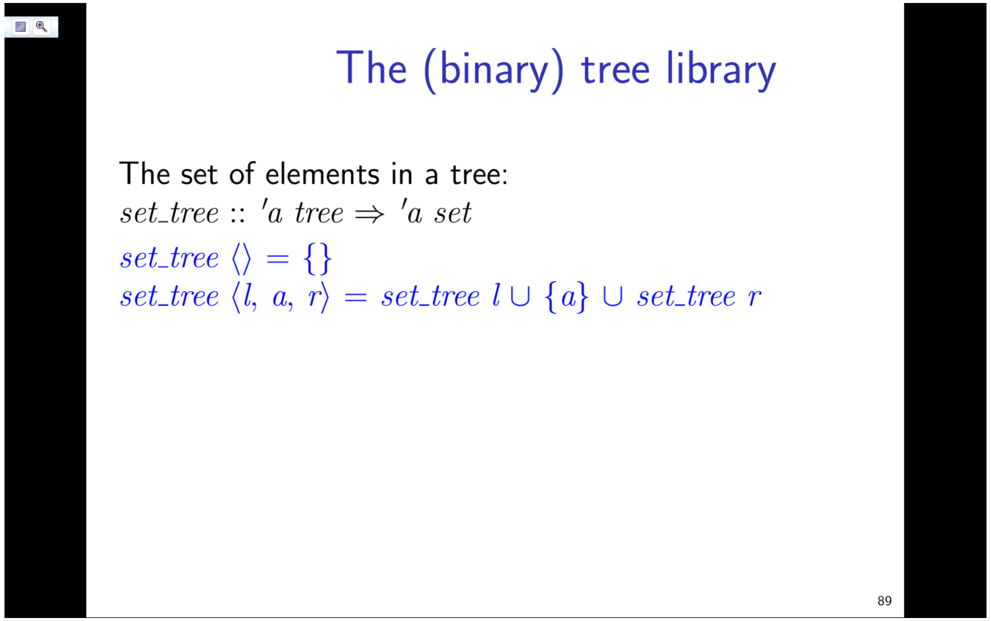
$size\ \langle l,\ \_,\ r\rangle = size\ l + size\ r + 1$



The (binary) tree library

The set of elements in a tree:

$set\_tree :: 'a\ tree \Rightarrow 'a\ set$



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Inorder listing:
$inorder :: 'a\ tree \Rightarrow 'a\ list$

$inorder\ \langle\rangle = []$
$inorder\ \langle l,\ x,\ r\rangle = inorder\ l\ @\ [x]\ @\ inorder\ r$

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Binary search tree invariant:
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For any type $'a$?

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Example: $bst :: ('a :: linorder)\ tree \Rightarrow bool$

Case study

BST_Demo.thy

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BST_Demo.thy

Chapter 4

Logic and Proof Beyond Equality

5 Logical Formulas

6 Proof Automation

7 Single Step Proofs

Syntax (in decreasing precedence):

$$\begin{array}{lllll} \mathit{form} & ::= & (\mathit{form}) & \mid\ \mathit{term} = \mathit{term} & \mid\ \neg \mathit{form} \\ & \mid & \mathit{form} \wedge \mathit{form} & \mid\ \mathit{form} \vee \mathit{form} & \mid\ \mathit{form} \longrightarrow \mathit{form} \\ & \mid & \forall x.\ \mathit{form} & \mid\ \exists x.\ \mathit{form} & \end{array}$$

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Input syntax: $\longleftrightarrow$ (same precedence as $\longrightarrow$)

Variable binding convention:

$$\forall\, x\ y.\ P\ x\ y\ \equiv\ \forall\, x.\ \forall\, y.\ P\ x\ y$$

Warning

Quantifiers have low precedence
and need to be parenthesized (if in some context)

$$\textbf{!}\quad P \wedge \forall\, x.\ Q\ x\ \rightsquigarrow\ P \wedge (\forall\, x.\ Q\ x)\quad \textbf{!}$$

Mathematical symbols

. . . and their ascii representations:

$\forall$	`\<forall>`	`ALL`
$\exists$	`\<exists>`	`EX`
λ	`\<lambda>`	`%`
$\longrightarrow$	`-->`	
$\longleftrightarrow$	`<->`	
$\wedge$	`/\`	`&`
$\vee$	`\/`	`|`
$\neg$	`\<not>`	`~`
$\neq$	`\<noteq>`	`~=`

Sets over type $'a$

$'a\ set$

- $\{\},\quad \{e_1,\ldots,e_n\}$

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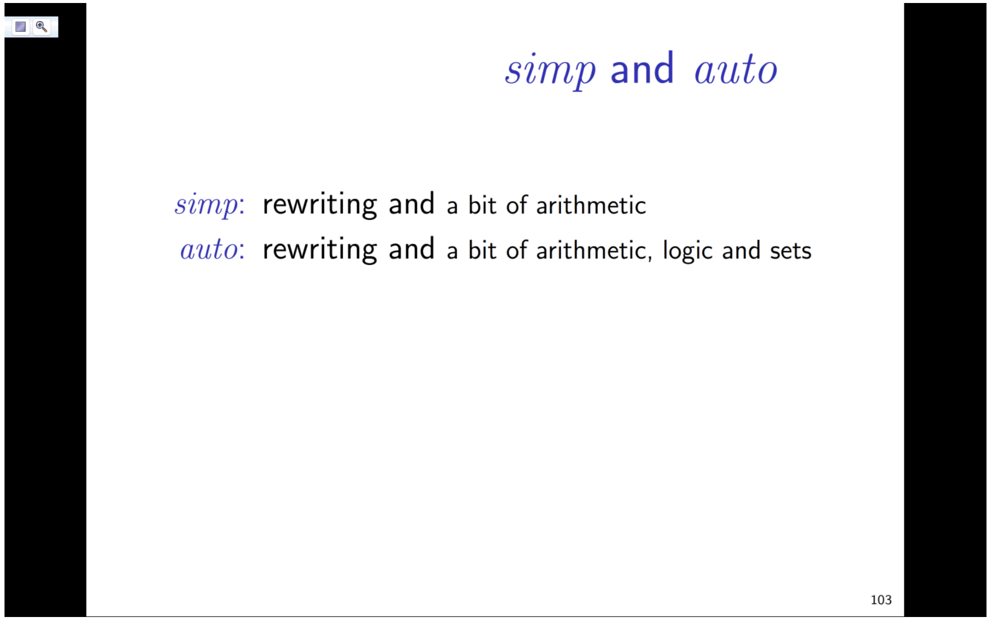
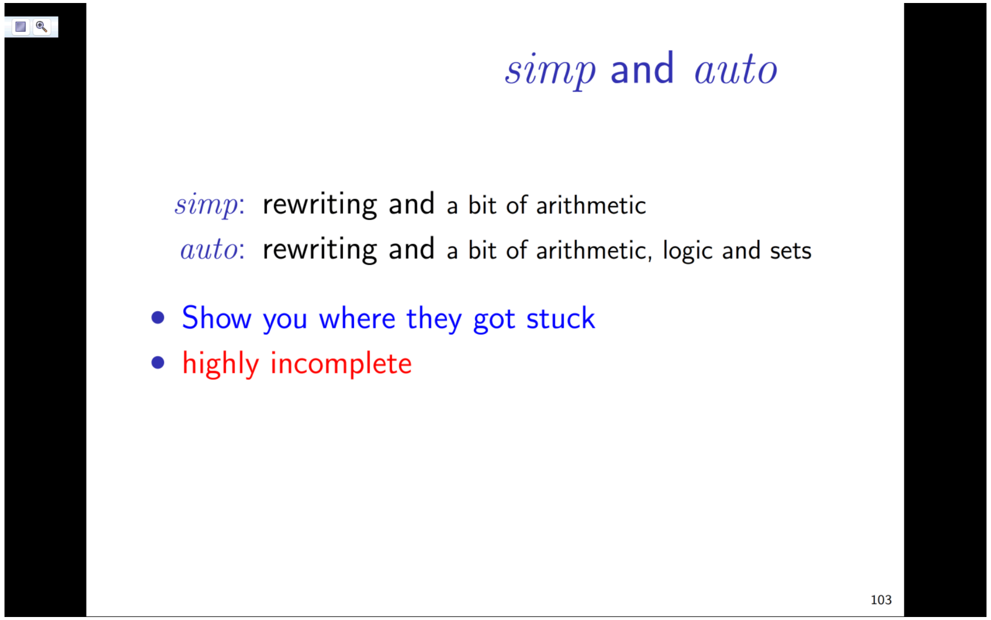


simp and *auto*

simp: rewriting and a bit of arithmetic

auto: rewriting and a bit of arithmetic, logic and sets

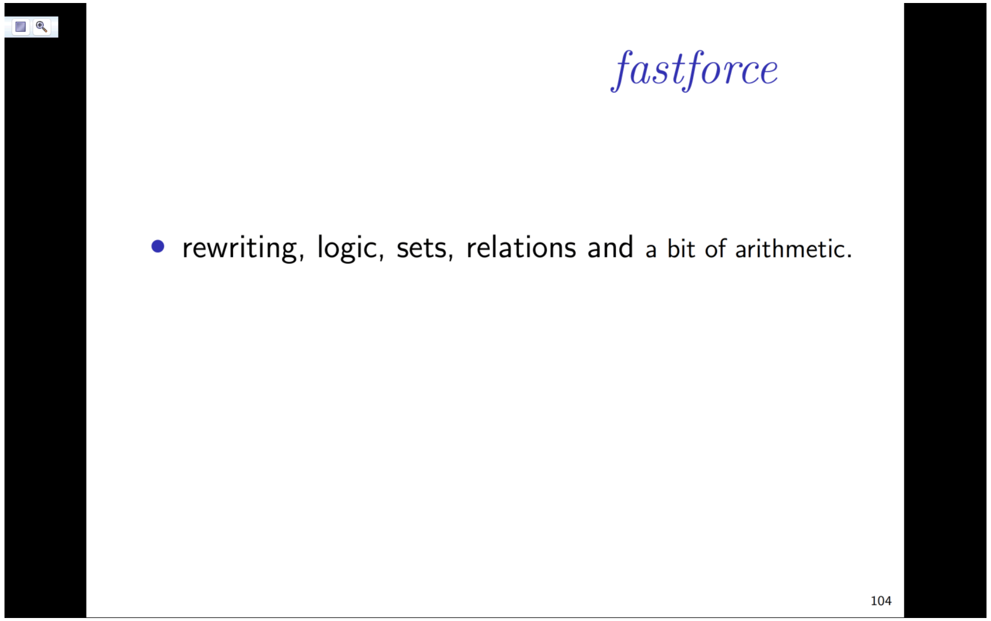


simp and *auto*

simp: rewriting and a bit of arithmetic

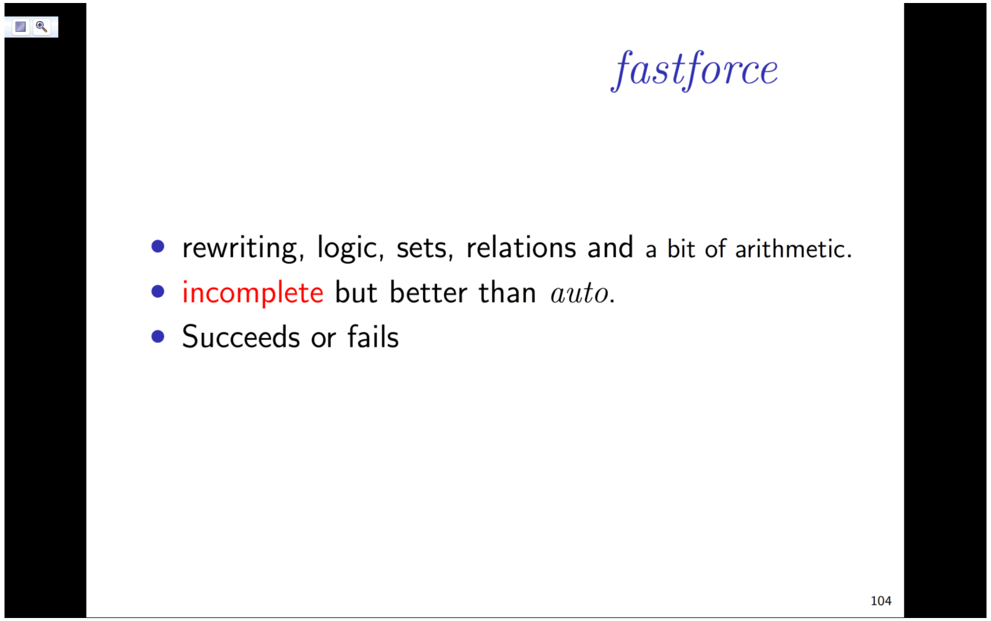
auto: rewriting and a bit of arithmetic, logic and sets

- Show you where they got stuck
- highly incomplete



fastforce

- rewriting, logic, sets, relations and a bit of arithmetic.



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- incomplete but better than *auto*.
- Succeeds or fails

fastforce

- rewriting, logic, sets, relations and a bit of arithmetic.
- incomplete but better than *auto*.
- Succeeds or fails
- Extensible with new *simp*-rules

blast

- A complete proof search procedure for FOL . . .
- . . . but (almost) without "="

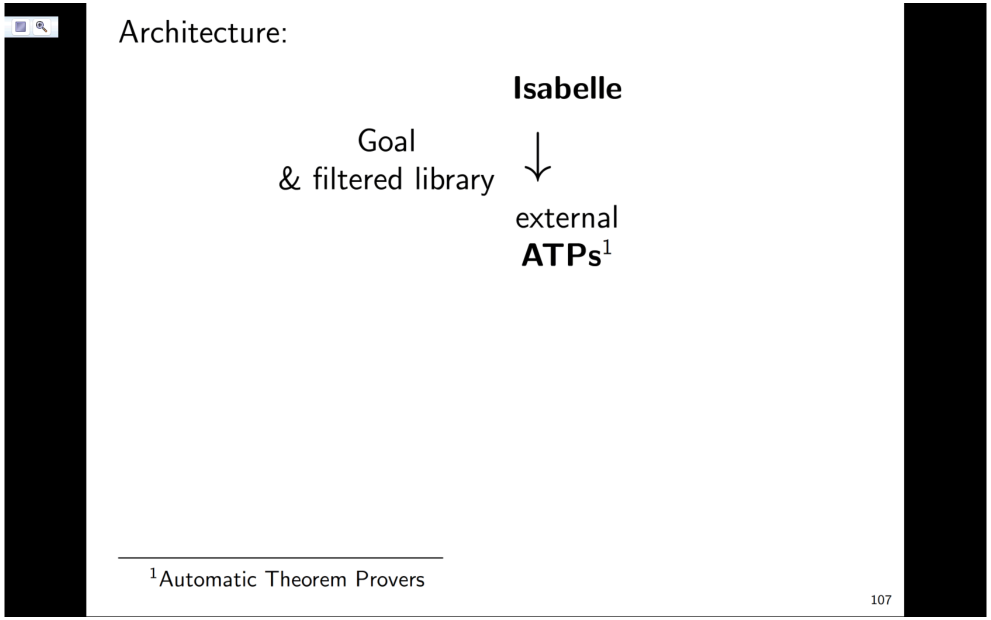
Sledgehammer

Architecture:

Isabelle

external
ATPs[1]

[1] Automatic Theorem Provers



Architecture:

Isabelle

Goal
& filtered library ↓

external
ATPs[1]

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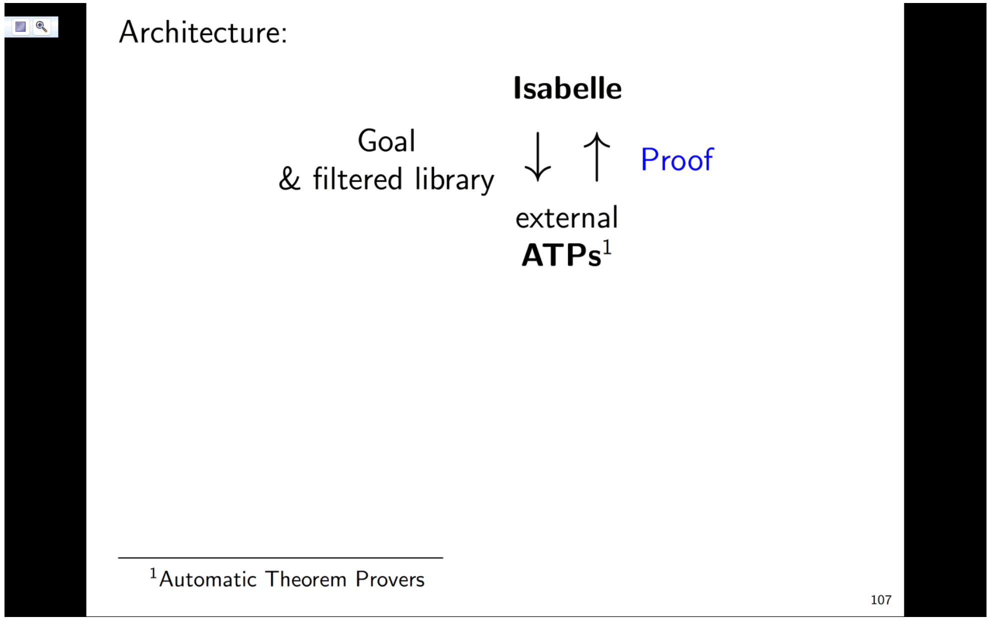


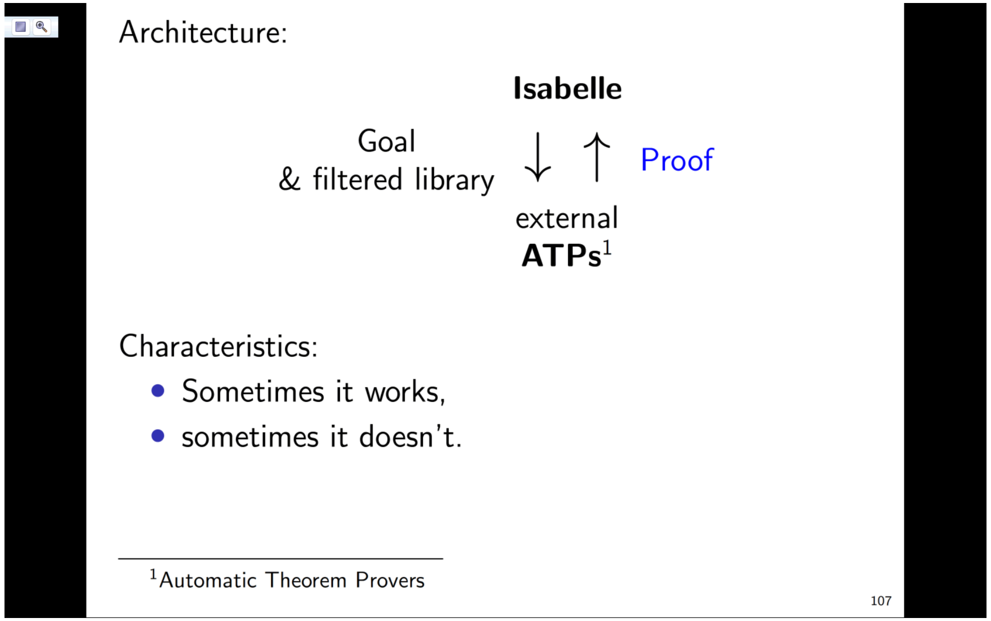
Architecture:

Isabelle

Goal
& filtered library ↓ ↑ Proof

external
ATPs[1]

[1] Automatic Theorem Provers



Architecture:

Isabelle

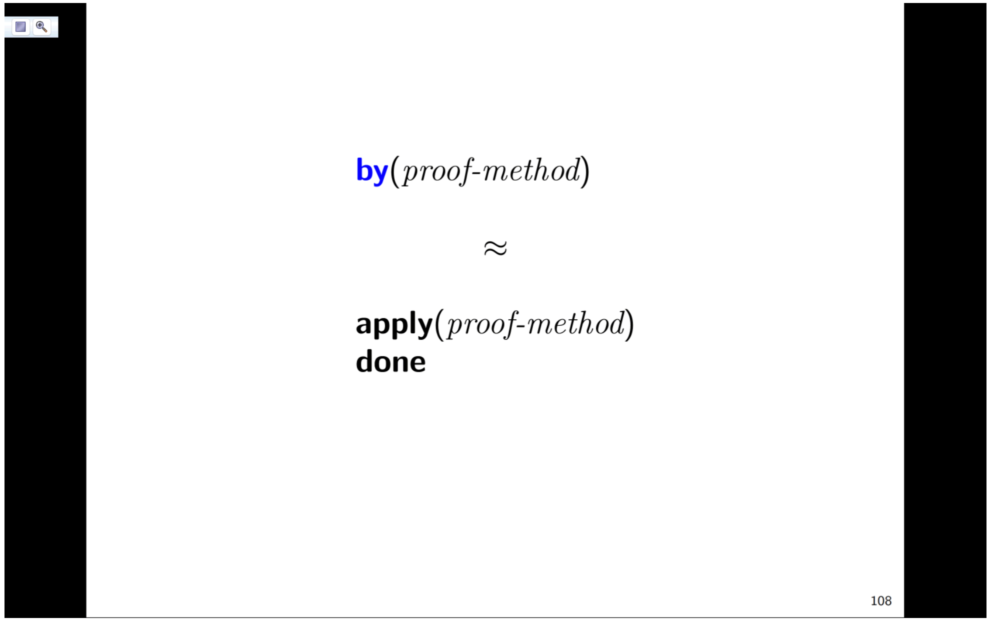
Goal
& filtered library ↓ ↑ Proof

external
ATPs[1]

Characteristics:

- Sometimes it works,
- sometimes it doesn't.

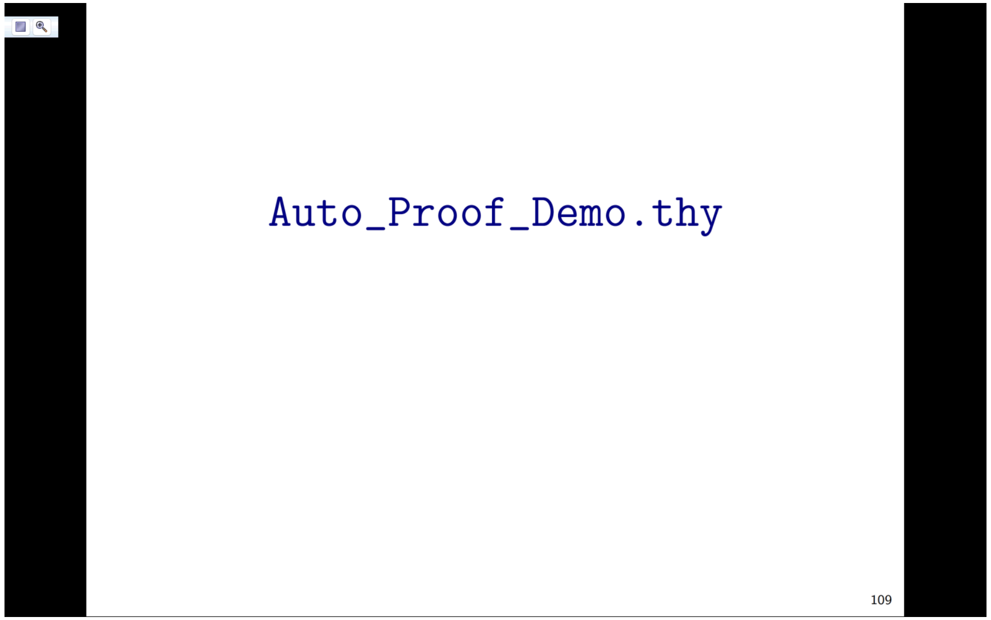
[1] Automatic Theorem Provers



by(*proof-method*)

≈

apply(*proof-method*)
done



Auto_Proof_Demo.thy