Script **generated by TTT**

Title: FDS (02.06.2017)

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TTT 100% Fri 08:30

When automation fails

Split proof up into smaller steps.

maximize

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- Better: convert to structured proof

Local lemmas

have B **if** *name:* $A_1 \ \dots \ A_m$ **for** $x_1 \ \dots \ x_n$

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proves $\llbracket A_1; \dots ; A_m \rrbracket \Longrightarrow B$
where all x_i have been replaced by $?x_i$.

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In general: **proof** *method*

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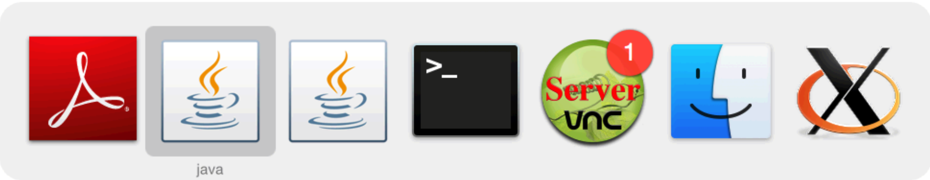
Separated by **next**

Isar_Induction_Demo.thy

Proof by cases

Isar_Induction_Demo.thy



Datatype case analysis

datatype $t = C_1\ \vec{\tau}\ \mid\ \ldots$

$$\boxed{\begin{array}{l}\textbf{proof}\ (cases\ "term")\\ \quad \textbf{case}\ (C_1\ x_1\ \ldots\ x_k)\\ \quad \ldots\ x_j\ \ldots\\ \textbf{next}\\ \vdots\\ \textbf{qed}\end{array}}$$

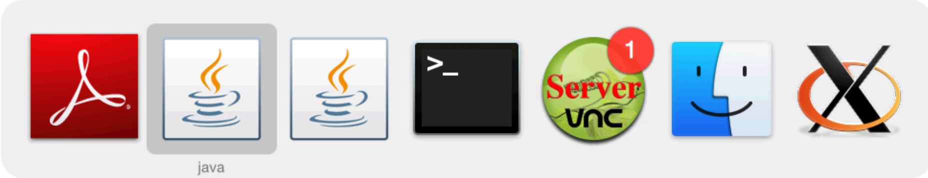
where $\quad \textbf{case}\ (C_i\ x_1\ \ldots\ x_k) \quad \equiv$

$$\begin{array}{l}\textbf{fix}\ x_1\ \ldots\ x_k\\ \textbf{assume}\ \underbrace{C_i\text{:}}_{\text{label}}\ \underbrace{term = (C_i\ x_1\ \ldots\ x_k)}_{\text{formula}}\end{array}$$

`Isar_Induction_Demo.thy`

Structural induction for nat

`Isar_Induction_Demo.thy`



Structural induction for nat

show $P(n)$
proof $(induction\ n)$
  case 0 $\equiv$ **let** $?case = P(0)$
  $\vdots$
  show $?case$
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Structural induction with $\Longrightarrow$

show $A(n) \Longrightarrow P(n)$
proof $(induction\ n)$
  case 0 $\equiv$ **assume** 0: $A(0)$
  ⋮ **let** $?case = P(0)$
  show $?case$
next
  case $(Suc\ n)$ $\equiv$ **fix** n
  ⋮ **assume** Suc: $A(n) \Longrightarrow P(n)$
  $A(Suc\ n)$
  ⋮ **let** $?case = P(Suc\ n)$
  show $?case$
qed

Named assumptions

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$$A_1 \Longrightarrow \ldots \Longrightarrow A_n \Longrightarrow B$$

by structural induction:
In the context of

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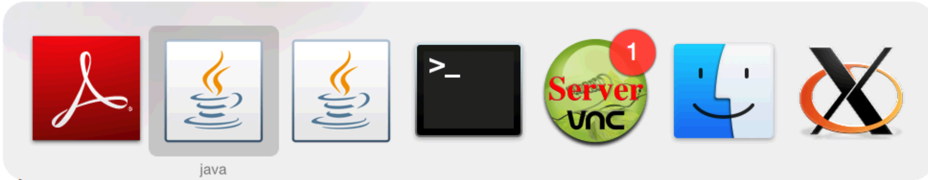

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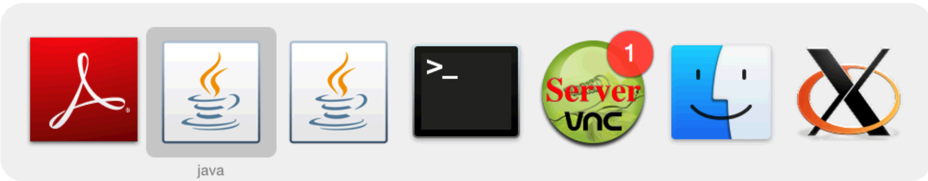
`Isar_Induction_Demo.thy`



Computation induction

If function f is defined by **fun** with n equations:

proof$(induction\ s\ t\ \ldots\ rule\text{:}\ f.induct)$

Generates cases named $i = 1 \ldots n$:

case $(i\ x\ y\ \ldots)$

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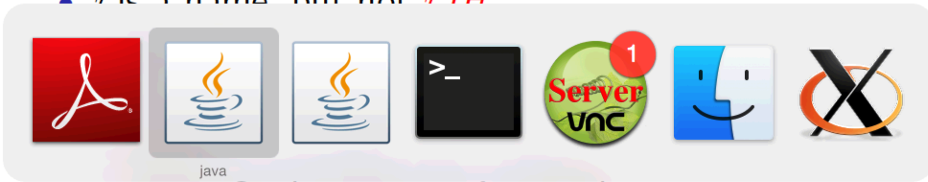
Isabelle/jEdit generates Isar template for you!

Computation induction

Naming

- i is a name, but not $i.IH$



$\leadsto$ Isabelle instantiates overlapping equations
$\leadsto$ case names of the form "i_j"

$sorted :: ('a{::}linorder)\ list \Rightarrow bool$

$sorted\ [] = True$
$sorted\ (x \# xs) = ((\forall y{\in}set\ xs.\ x \leq y) \wedge sorted\ xs)$

Correctness of sorting

Specification of $sort :: ('a{::}linorder)\ list \Rightarrow 'a\ list$:

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Is that it? How about

$$set\ (sort\ xs) = set\ xs$$

Better: every x occurs as often in $sort\ xs$ as in xs.

More succinctly:

$$mset\ (sort\ xs) = mset\ xs$$

where $mset :: 'a\ list \Rightarrow 'a\ multiset$

What are multisets?

Sets with (possibly) repeated elements

Some operations:

$$\begin{array}{rcl} \{\#\} & :: & 'a\ multiset \\ add\_mset & :: & 'a \Rightarrow 'a\ multiset \Rightarrow 'a\ multiset \\ + & :: & 'a\ multiset \Rightarrow 'a\ multiset \Rightarrow 'a\ multiset \end{array}$$

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1. Correctness
2. Insertion Sort
3. Time
4. Merge Sort

Thys/Sorting.thy

Insertion sort correctness