Script **generated by TTT**

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datatype — the general case

$$\textbf{datatype}\ (\alpha_1, \ldots, \alpha_n)t \;=\; C_1\ \tau_{1,1} \ldots \tau_{1,n_1} \;\mid\; \ldots \;\mid\; C_k\ \tau_{k,1} \ldots \tau_{k,n_k}$$

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Distinctness and injectivity are applied automatically
Induction must be applied explicitly

Case expressions

Like in functional languages:

$$(\mathit{case}\ t\ \mathit{of}\ \ \mathit{pat}_1 \Rightarrow t_1 \mid \dots \mid \mathit{pat}_n \Rightarrow t_n)$$

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Need ( ) in context

`Tree_Demo.thy`

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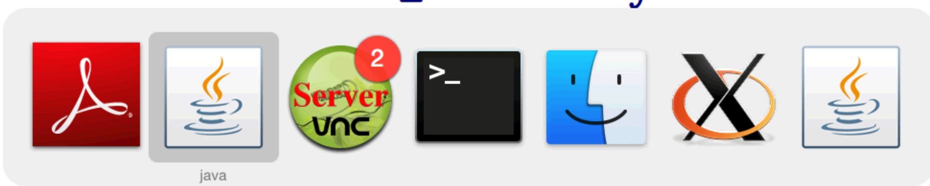

The *option* type

datatype $'a\ option = None \mid Some\ 'a$

If $'a$ has values $a_1,\ a_2,\ \ldots$
then $'a\ option$ has values $None,\ Some\ a_1,\ Some\ a_2,\ \ldots$

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2 Type and function definitions

Type definitions

Function definitions

Non-recursive definitions

Example

definition $sq :: nat \Rightarrow nat$ **where** $sq\ n \ = \ n{*}n$

Non-recursive definitions

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The danger of nontermination

How about $f\ x = f\ x + 1$?

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How about $f\ x = f\ x + 1$?

Subtract $f\ x$ on both sides.
$\Longrightarrow 0 = 1$

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- Termination must be provable automatically by size measures
- Proves customized induction schema

Example: separation

fun $sep :: 'a \Rightarrow 'a\ list \Rightarrow 'a\ list$ **where**
$sep\ a\ (x\#y\#zs) = x\ \#\ a\ \#\ sep\ a\ (y\#zs)\ \ |$
$sep\ a\ xs = xs$

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Basic induction heuristics

Theorems about recursive functions
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Induction on argument number i of f
if f is defined by recursion on argument number i

A tail recursive reverse

Our initial reverse:

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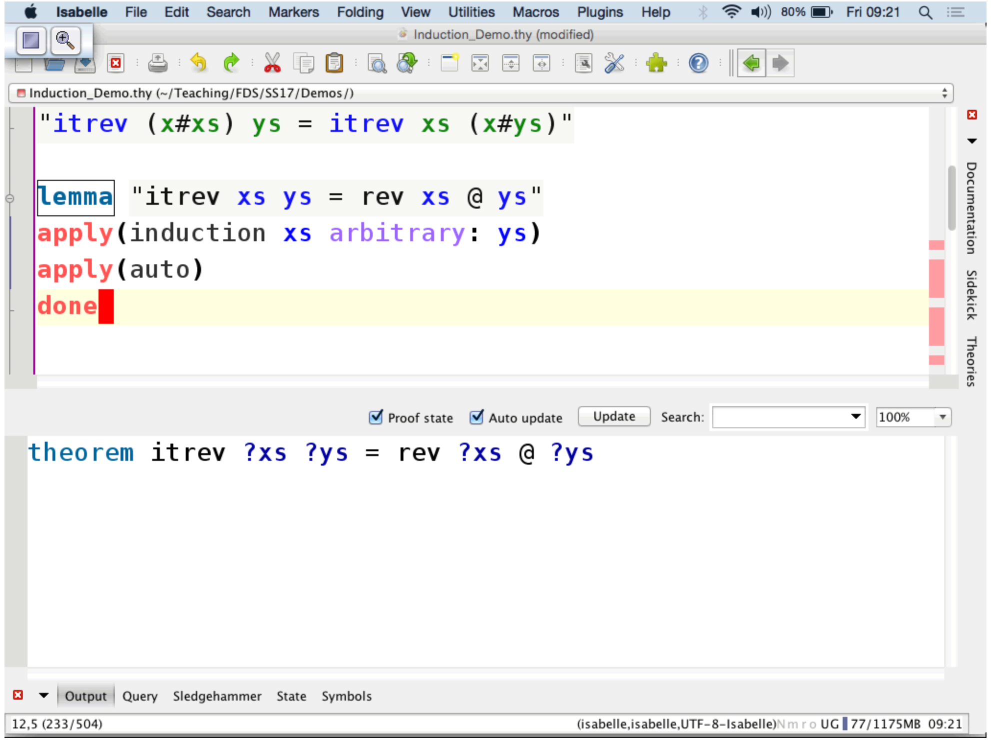
lemma $itrev\ xs\ [] = rev\ xs$

Induction_Demo.thy



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Now: induction for complex recursion patterns.

Computation Induction

Example

fun $div2 :: nat \Rightarrow nat$ **where**
$div2\ 0 = 0\ \ |$
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$\leadsto$ induction rule `div2.induct`:

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- ideally the a_i should be variables.

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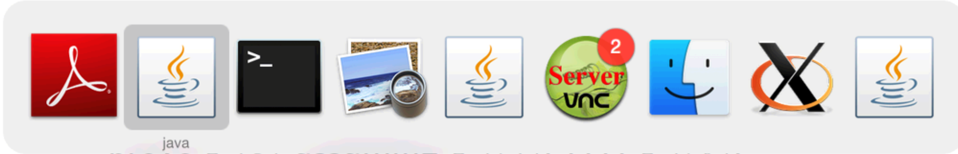

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Simplification = (Term) Rewriting

An example

Equations:

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\begin{aligned}
0 + n &= n & (1)\\
(Suc\ m) + n &= Suc\ (m + n) & (2)\\
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(0 \leq m) &= True & (4)
\end{aligned}
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$$[\![P_1; \ldots; P_k]\!] \Longrightarrow l = r$$

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We can simplify $f(0)$ to $g(0)$ but
we cannot simplify $f(1)$ because $p(1)$ is not provable.

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$$n < m \Longrightarrow (n < Suc\ m) = True \quad \text{YES}$$
$$Suc\ n < m \Longrightarrow (n < m) = True \quad \text{NO}$$

Proof method *simp*

Goal: 1. $[\![P_1; \ldots; P_m]\!] \Longrightarrow C$

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- rules from **fun** and **datatype**

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Variations:

- $(simp\ \ldots\ del{:}\ \ldots)$ removes *simp*-lemmas
- *add* and *del* are optional

auto versus *simp*

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auto versus *simp*

- *auto* acts on all subgoals
- *simp* acts only on subgoal 1
- *auto* applies *simp* and more
- *auto* can also be modified:
  $(\mathit{auto\ simp\ add}{:}\ \ldots\ \mathit{simp\ del}{:}\ \ldots)$

Rewriting with definitions

Definitions (**definition**) must be used explicitly:

$$(\mathit{simp\ add}{:}\ \mathit{f\_def}\ \ldots)$$

Case splitting with *simp*/*auto*

Automatic:

$$P(\mathit{if}\ A\ \mathit{then}\ s\ \mathit{else}\ t)$$
$$=$$
$$(A \longrightarrow P(s)) \wedge (\neg A \longrightarrow P(t))$$

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Proof method: *(simp split: nat.split)*

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Proof method: *(simp split: nat.split)*
Or *auto*. Similar for any datatype *t*: *t.split*

Case splitting with *simp/auto*

Automatic:

$$P(\textit{if } A \textit{ then } s \textit{ else } t) \\ = \\ (A \longrightarrow P(s)) \wedge (\neg A \longrightarrow P(t))$$

By hand:

$$P(\textit{case } e \textit{ of } 0 \Rightarrow a \mid \textit{Suc } n \Rightarrow b) \\ = \\ (e = 0 \longrightarrow P(a)) \wedge (\forall n.\ e = \textit{Suc } n \longrightarrow P(b))$$

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Or *auto*. Similar for any datatype *t*: *t.split*

Splitting pairs with *simp/auto*

How to replace

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Proof method: *(simp split: prod.split)*

`Simp_Demo.thy`