Script **generated by TTT**

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Chapter 4:
Bottom-up Analysis

Bottom-up Analysis

Attention:
Many grammars are not $LL(k)$!

A reason for that is:

Definition
Grammar G is called left-recursive, if

$$\boxed{A} \rightarrow^{+} \boxed{A\,\beta} \quad \text{for an} \quad A \in N,\ \beta \in (T \cup N)^*$$

Bottom-up Analysis

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$$A \rightarrow^{+} A\,\beta \quad \text{for an} \quad A \in N,\ \beta \in (T \cup N)^*$$

Example:

$$\begin{array}{lll} \boxed{E \rightarrow E} + T & \mid & T \\ \boxed{T \rightarrow T} * F & \mid & F \\ F \rightarrow (\,E\,) & \mid & \text{name} \quad \mid \quad \text{int} \end{array}$$

... is left-recursive

Bottom-up Analysis

Theorem:

Let a grammar G be reduced and left-recursive, then G is not $LL(k)$ for any k.

Proof:

Let $A \boxed{\rightarrow} A\,\beta \mid \alpha \quad \in P$
and A be reachable from S

Assumption: G is $LL(k)$

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$\Rightarrow \text{First}_k(\alpha\,\beta^n\,\gamma) \cap$
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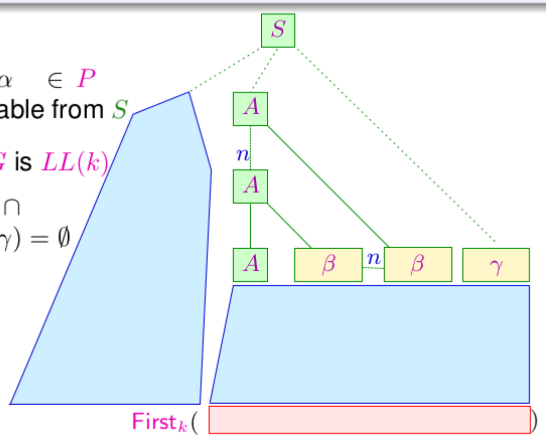

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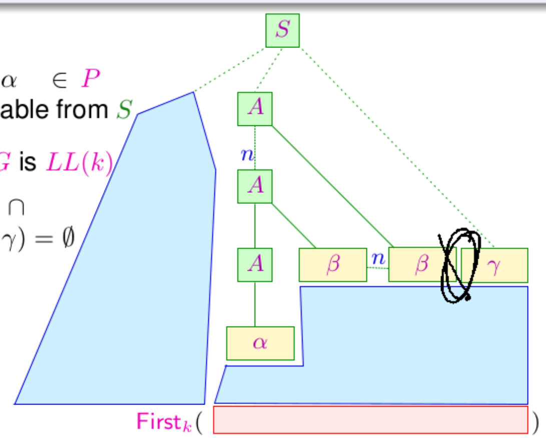

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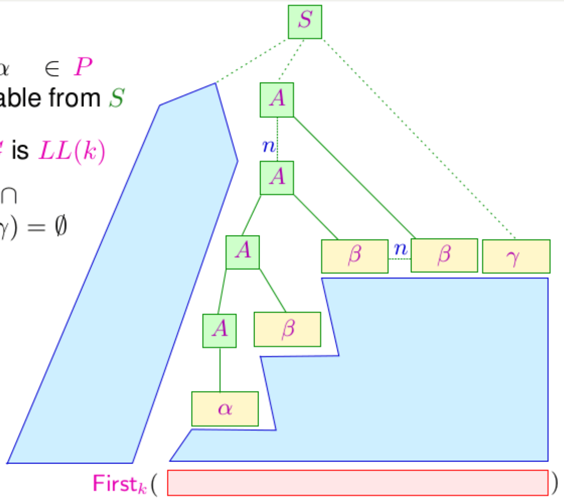

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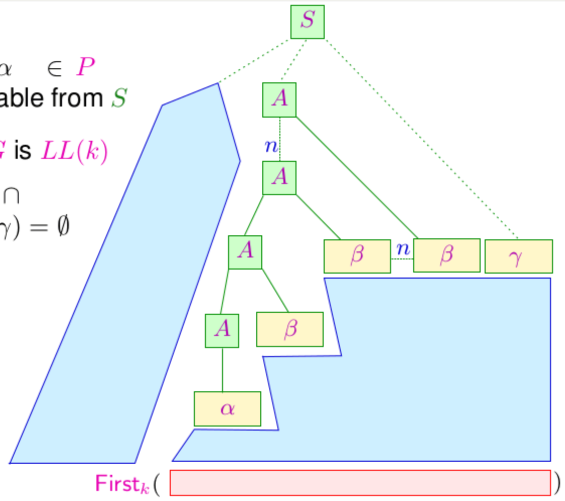

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$\Rightarrow \text{First}_k(\alpha\,\beta^n\,\gamma)\,\cap$ $\rightarrow \alpha\gamma$
$\text{First}_k(\alpha\,\beta^{n+1}\,\gamma) = \emptyset$ $\rightarrow \alpha\gamma$

Case 1: $\beta \to^* \epsilon$ — Contradiction !!!
Case 2: $\beta \to^* w \neq \epsilon \implies \text{First}_k(\alpha\,w^k\,\gamma) \cap \text{First}_k(\alpha\,w^{k+1}\,\gamma) \neq \emptyset$

Shift-Reduce Parser


Donald Knuth

Idea:

We *delay* the decision whether to reduce until we know, whether the input matches the right-hand-side of a rule!

Construction: Shift-Reduce parser M_G^R

- The input is shifted successively to the pushdown.
- Is there a complete right-hand side (a handle) atop the pushdown, it is replaced (reduced) by the corresponding left-hand side

Shift-Reduce Parser

Example:

$$\begin{array}{lcl} S & \to & A\,B \\ A & \to & a \\ B & \to & b \end{array}$$

The pushdown automaton:

States: q_0, f, a, b, A, B, S;
Start state: q_0
End state: f

q_0	a	$q_0\,a$
a	ϵ	A
A	b	$A\,b$
b	ϵ	B
$A\,B$	ϵ	S
$q_0\,S$	ϵ	f

Shift-Reduce Parser

Construction:

In general, we create an automaton $M_G^R = (Q, T, \delta, q_0, F)$ with:

- $Q = T \cup N \cup \{q_0, f\}$ $\quad$ (q_0, f fresh);
- $F = \{f\}$;
- Transitions:

$$\begin{array}{lcll} \delta & = & \{(q, x, q\,x) \mid q \in Q, x \in T\} \;\cup & // \text{ Shift-transitions} \\ & & \{(q\,\alpha, \boxed{\epsilon,} q\,A) \mid q \in Q, A \to \alpha \in P\} \;\cup & // \text{ Reduce-transitions} \\ & & \{(q_0\,S, \boxed{\epsilon,} f)\} & // \text{ finish} \end{array}$$

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Example-computation:

$$\begin{array}{lclcl} (q_0, \; a\,b) & \vdash & (q_0\,a, \; b) & \vdash & (q_0\,A, \; b) \\ & \vdash & (q_0\,A\,b, \; \epsilon) & \vdash & (q_0\,A\,B, \; \epsilon) \\ & \vdash & (q_0\,S, \; \epsilon) & \vdash & (f, \; \epsilon) \end{array}$$

Shift-Reduce Parser

Observation:

- The sequence of reductions corresponds to a reverse rightmost-derivation for the input
- To prove correctnes, we have to prove:

$$\boxed{(\epsilon, \; w)} \vdash^* \boxed{(A, \; \epsilon)} \quad \text{iff} \quad \boxed{A \to^* w}$$

- The shift-reduce pushdown automaton M_G^R is in general also non-deterministic
- For a deterministic parsing-algorithm, we have to identify computation-states for reduction

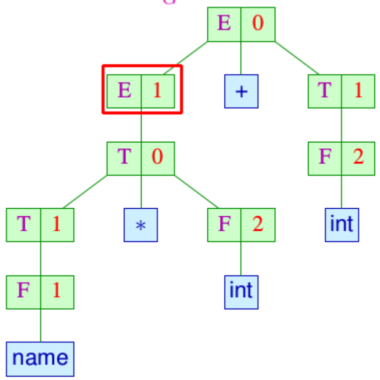
$\Longrightarrow$ LR-Parsing

Reverse Rightmost Derivations in Shift-Reduce-Parsers

Idea: Observe *reverse rightmost*-derivations of M_G^R!

Input:
counter $* \; 2 + 40$

Pushdown:
$(\; q_0 \;)$



Reverse Rightmost Derivations in Shift-Reduce-Parsers

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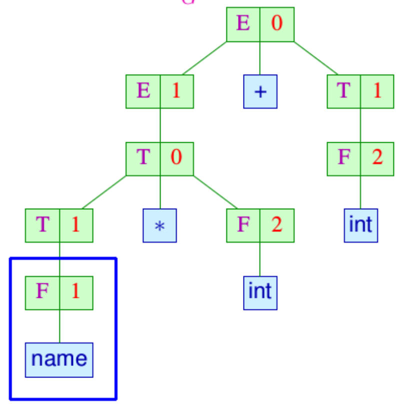
Input:
$* \; 2 + 40$

Pushdown:
$(\; q_0 \; name \;)$

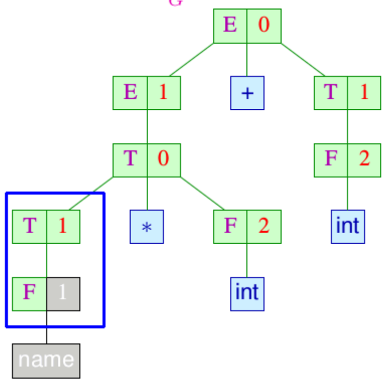


Reverse Rightmost Derivations in Shift-Reduce-Parsers

Idea: Observe *reverse rightmost*-derivations of M_G^R!

Input:
$* \; 2 + 40$

Pushdown:
$(\; q_0 \; F \;)$



Reverse Rightmost Derivations in Shift-Reduce-Parsers

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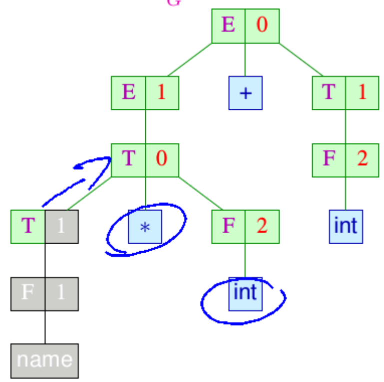
Input:
$* \; 2 + 40$

Pushdown:
$(\; q_0 \; T \;)$



Reverse Rightmost Derivations in Shift-Reduce-Parsers

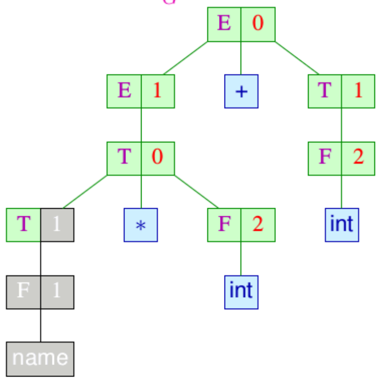
Idea: Observe *reverse rightmost*-derivations of M_G^R!

Input:

$+ \; 40$

Pushdown:

$(\; q_0 \; T * int \;)$



Reverse Rightmost Derivations in Shift-Reduce-Parsers

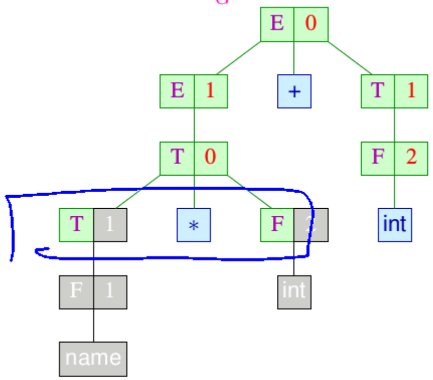
Idea: Observe *reverse rightmost*-derivations of M_G^R!

Input:

$+ \; 40$

Pushdown:

$(\; q_0 \; \boxed{T * F} \;)$



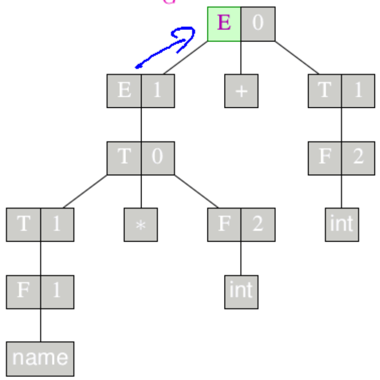
Reverse Rightmost Derivations in Shift-Reduce-Parsers

Idea: Observe *reverse rightmost*-derivations of M_G^R!

Input:

Pushdown:

$(\; f \;)$



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Input:

$+ \; 40$

Pushdown:

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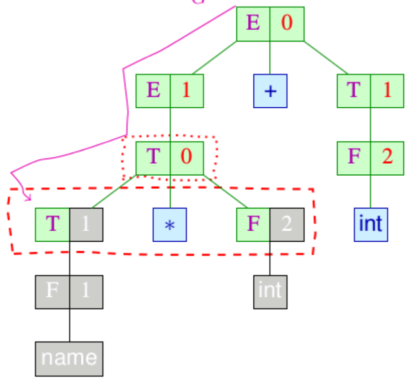

Generic Observation:

In a sequence of configurations of M_G^R

$$(q_0 \, \alpha \, \gamma, \; v) \vdash (q_0 \, \alpha \, B, \; v) \vdash^* (q_0 \, S, \; \epsilon)$$

we call $\alpha \, \gamma$ a viable prefix for the complete item $[B \to \gamma \bullet]$.

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Idea: Observe *reverse rightmost*-derivations of M_G^R!

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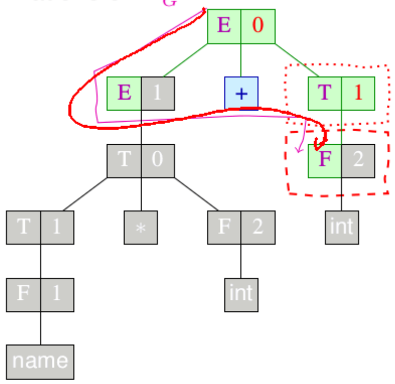


Pushdown:
$(q_0\, E + F)$

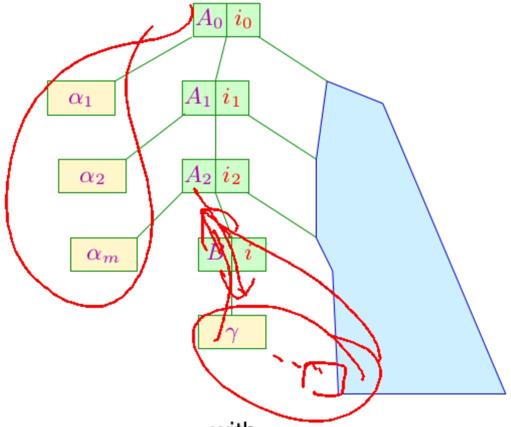
Generic Observation:
In a sequence of configurations of M_G^R

$$(q_0\, \alpha\, \gamma,\ v) \vdash (q_0\, \alpha\, B,\ v) \vdash^* (q_0\, S,\ \epsilon)$$

we call $\alpha\,\gamma$ a viable prefix for the complete item $[B \to \gamma\bullet]$.

Bottom-up Analysis: Viable Prefix

$\alpha\,\gamma$ is viable for $[B \to \gamma\bullet]$ iff $S \to_R^* \alpha\, B\, v$



... with $\alpha = \alpha_1 \ldots \alpha_m$

Bottom-up Analysis: Viable Prefix

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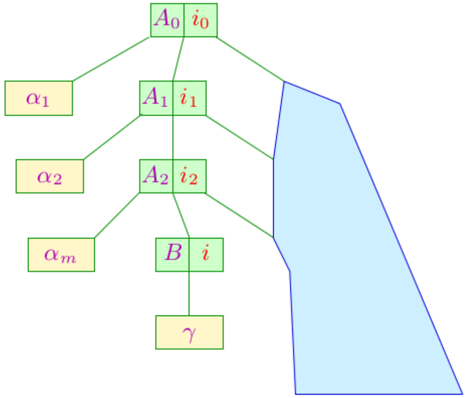
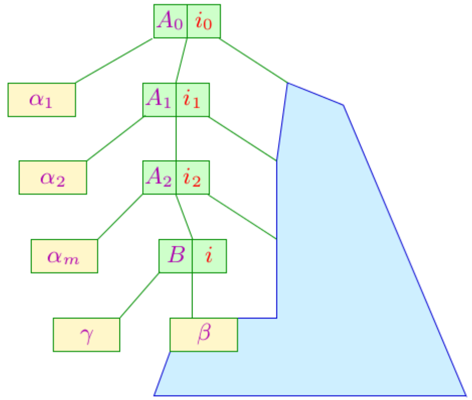


... with $\alpha = \alpha_1 \ldots \alpha_m$

Conversely, for an arbitrary valid word α' we can determine the set of all later on possibly matching rules ...

Bottom-up Analysis: Admissible Items

The item $[B \to \gamma \bullet \beta]$ is called admissible for α' iff $S \to_R^* \alpha\, B\, v$ with $\alpha' = \alpha\,\gamma$:



... with $\alpha = \alpha_1 \ldots \alpha_m$

Characteristic Automaton

Observation:

The set of viable prefixes from $(N \cup T)^*$ for (admissible) items can be computed from the content of the shift-reduce parser's pushdown with the help of a finite automaton:

States: Items

Start state: $[S' \to \bullet\, S]$

Final states: $\{[B \to \gamma \bullet] \mid B \to \gamma \in P\}$

Transitions:

(1) $([A \to \alpha \bullet X\, \beta], X, [A \to \alpha\, X \bullet \beta])$, $\quad X \in (N \cup T), A \to \alpha\, X\, \beta \in P$;

(2) $([A \to \alpha \bullet B\, \beta], \epsilon, [B \to \bullet\, \gamma])$, $\quad A \to \alpha\, B\, \beta\,,\; B \to \gamma \in P$;

The automaton $c(G)$ is called characteristic automaton for G.

Reverse Rightmost Derivations in Shift-Reduce-Parsers

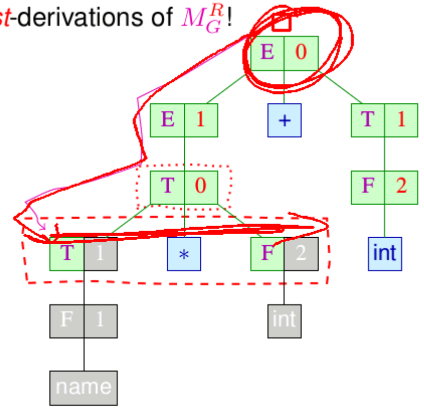
Idea: Observe *reverse rightmost*-derivations of M_G^R!

Input:

$+\ 40$

Pushdown:

$(\ q_0\ T * F\ )$



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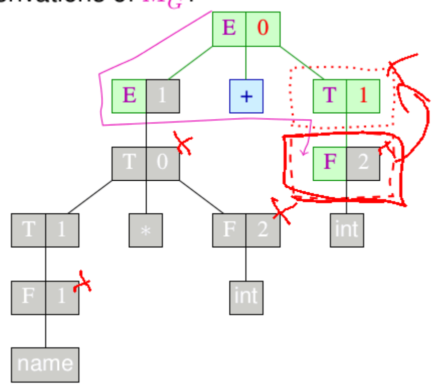
Pushdown:

$(\ q_0\ E + F\ )$



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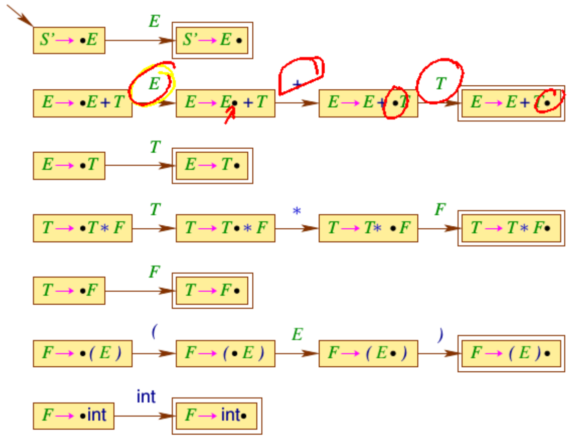
Characteristic Automaton

For example:

$$\begin{array}{lcl} E & \to & E + T \quad \mid \quad T \\ T & \to & T * F \quad \mid \quad F \\ F & \to & (\ E\ ) \quad \mid \quad \text{int} \end{array}$$



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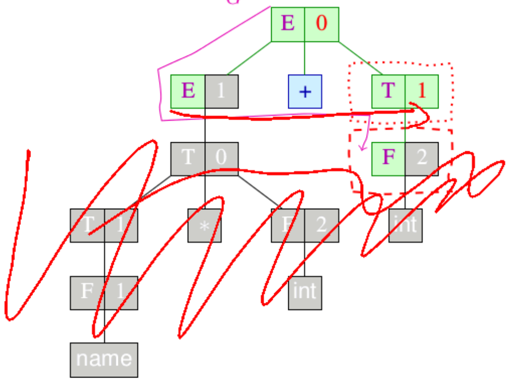

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$+\ 40$

Pushdown:
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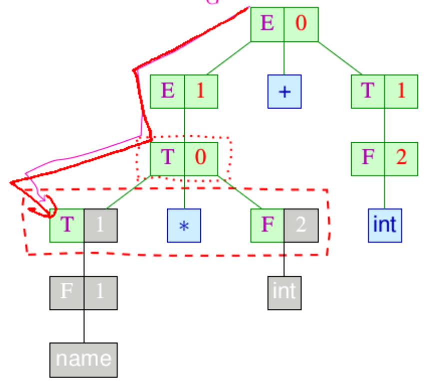

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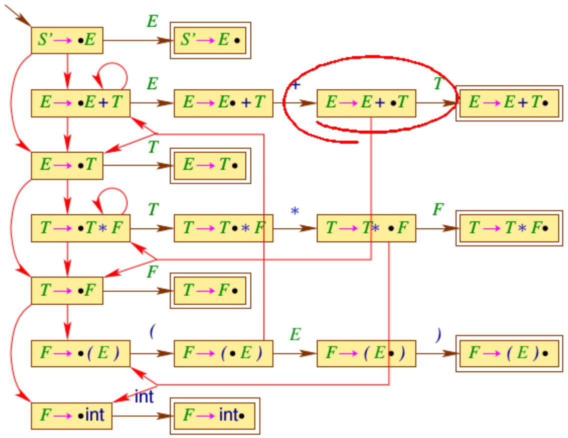
F & \to & (\ E\ ) \quad | \quad \text{int}
\end{array}
$$

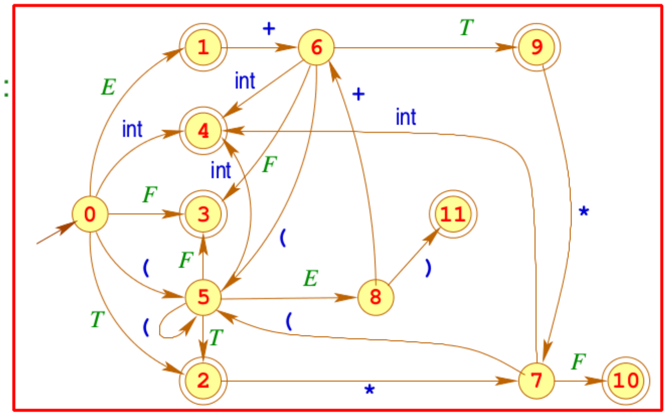


Canonical LR(0)-Automaton

The canonical $LR(0)$-automaton $LR(G)$ is created from $c(G)$ by:

1. performing arbitrarily many ϵ-transitions after every consuming transition
2. performing the powerset construction

... for example:



Canonical LR(0)-Automaton

Example:

$$\begin{array}{lcll} E & \to & E+T & \mid\ T \\ T & \to & T*F & \mid\ F \\ F & \to & (\ E\ ) & \mid\ \mathsf{int} \end{array}$$

Therefore we determine:

$$\boxed{q_0} = \{ S' \to \bullet E,\ E \to \bullet E + T,\ E \to \bullet T,\ T \to \bullet T * F,\ T \to \bullet F,\ F \to \bullet (E),\ F \to \bullet\, \mathsf{int} \}$$

$$q_1 \neq q\,\delta(q_0, T) = \{ E \to T \bullet,\ T \to T \bullet * F \}$$

$$q_2 = \delta(q_0, ( ) =$$

Canonical LR(0)-Automaton

Example:

$$\begin{array}{lcll} E & \to & E+T & \mid\ T \\ T & \to & T*F & \mid\ F \\ F & \to & (\ E\ ) & \mid\ \mathsf{int} \end{array}$$

Therefore we determine:

$$\begin{aligned} q_0 &= \{[S' \to \bullet\, E], \\ &\quad [E \to \bullet\, E + T], \\ &\quad [E \to \bullet\, T], \\ &\quad [T \to \bullet\, T * F], \\ &\quad [T \to \bullet\, F], \\ &\quad [F \to \bullet\, (\ E\ )], \\ &\quad [F \to \bullet\, \mathsf{int}]\} \end{aligned}$$

$$\begin{aligned} q_1 &= \delta(q_0, E) = \{[S' \to E \bullet], [E \to E \bullet + T]\} \\ q_2 &= \delta(q_0, T) = \{[E \to T \bullet], [T \to T \bullet * F]\} \\ q_3 &= \delta(q_0, F) = \{[T \to F \bullet]\} \\ q_4 &= \delta(q_0, \mathsf{int}) = \{[F \to \mathsf{int} \bullet]\} \end{aligned}$$

Canonical LR(0)-Automaton

$$\begin{aligned} q_5 &= \delta(q_0, (\,) = \{[F \to (\ \bullet E\ )], \\ &\quad [E \to \bullet\, E + T], \\ &\quad [E \to \bullet\, T], \\ &\quad [T \to \bullet\, T * F], \\ &\quad [T \to \bullet\, F], \\ &\quad [F \to \bullet\, (\ E\ )], \\ &\quad [F \to \bullet\, \mathsf{int}]\} \end{aligned}$$

$$\begin{aligned} q_6 &= \delta(q_1, +) = \{[E \to E + \bullet\, T], \\ &\quad [T \to \bullet\, T * F], \\ &\quad [T \to \bullet\, F], \\ &\quad [F \to \bullet\, (\ E\ )], \\ &\quad [F \to \bullet\, \mathsf{int}]\} \end{aligned}$$

$$\begin{aligned} q_7 &= \delta(q_2, *) = \{[T \to T * \bullet\, F], [F \to \bullet\, (\ E\ )], [F \to \bullet\, \mathsf{int}]\} \\ q_8 &= \delta(q_5, E) = \{[F \to (\ E \bullet\ )], [E \to E \bullet + T]\} \\ q_9 &= \delta(q_6, T) = \{[E \to E + T \bullet], [T \to T \bullet * F]\} \\ q_{10} &= \delta(q_7, F) = \{[T \to T * F \bullet]\} \\ q_{11} &= \delta(q_8,)\,) = \{[F \to (\ E\ ) \bullet]\} \end{aligned}$$

Canonical LR(0)-Automaton

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$$\begin{aligned} q_6 &= \delta(q_1, +) = \{[E \to E + \bullet\, T], \\ &\quad [T \to \bullet\, T * F], \\ &\quad [T \to \bullet\, F], \\ &\quad [F \to \bullet\, (\ E\ )], \\ &\quad [F \to \bullet\, \mathsf{int}]\} \end{aligned}$$

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$$\begin{array}{lcll} E & \to & E+T & \mid\ T \\ T & \to & T*F & \mid\ F \\ F & \to & (\ E\ ) & \mid\ \text{int} \end{array}$$

Therefore we determine:

$$\begin{array}{lcl} q_0 & = & \{[S' \to \bullet \boxed{E}], \\ & & [E \to \bullet\, E+T], \\ & & [E \to \bullet \boxed{T}], \\ & & [T \to \bullet\, T*F]\} \\ & & [T \to \bullet\, F], \\ & & [F \to \bullet\, (\ E\ )], \\ & & [F \to \bullet\, \text{int}]\} \end{array}$$

$$\begin{array}{lcccl} q_1 & = & \delta(q_0, \boxed{E}) & = & \{[S' \to E\bullet], \\ & & & & [E \to E \bullet + T]\} \\ q_2 & = & \delta(q_0, \boxed{T}) & = & \{[E \to T\bullet], \\ & & & & [T \to T \bullet * F]\} \\ q_3 & = & \delta(q_0, F) & = & \{[T \to F\bullet]\} \\ q_4 & = & \delta(q_0, \text{int}) & = & \{[F \to \text{int}\bullet]\} \end{array}$$

Canonical LR(0)-Automaton

Observation:

The canonical $LR(0)$-automaton can be created directly from the grammar.
Therefore we need a helper function δ^*_ϵ (ϵ-closure)

$$\delta^*_\epsilon(q) = q \cup \{[B \to \bullet\gamma] \mid \exists\, [A \to \alpha \bullet B'\beta'] \in q\,, \quad \beta \in (N \cup T)^* : \quad B' \to^* B\,\beta\}$$

We define:

- **States:** Sets of items;
- **Start state:** $\delta^*_\epsilon\,\{[S' \to \bullet\, S]\}$
- **Final states:** $\{q \mid \exists\, A \to \alpha \in P : \boxed{[A \to \alpha\bullet] \in q}\}$
- **Transitions:** $\delta(q, X) = \delta^*_\epsilon\,\{[A \to \alpha\, X \bullet \beta] \mid [A \to \alpha \bullet X\,\beta] \in q\}$

LR(0)-Parser

Idea for a parser:

- The parser manages a viable prefix $\alpha = X_1 \ldots X_m$ on the pushdown and uses $LR(G)$, to identify reduction spots.
- It can reduce with $A \to \gamma$, if $[A \to \gamma\,\bullet]$ is admissible for α

Optimization:

We push the states instead of the X_i in order not to process the pushdown's content with the automaton anew all the time.
Reduction with $A \to \gamma$ leads to popping the uppermost $|\gamma|$ states and continue with the state on top of the stack and input A.

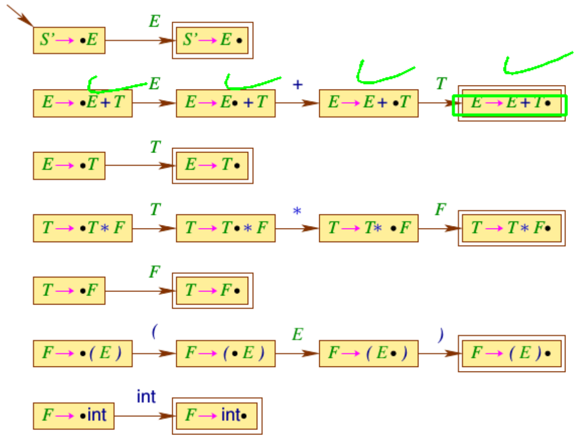
Attention:

This parser is only deterministic, if each final state of the canonical $LR(0)$-automaton is conflict free.

Characteristic Automaton

For example:

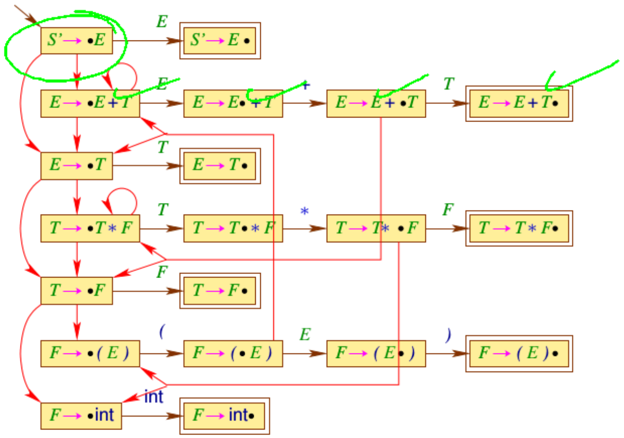
$$\begin{array}{lcll} E & \to & E+T & \mid\ T \\ T & \to & T*F & \mid\ F \\ F & \to & (\ E\ ) & \mid\ \text{int} \end{array}$$



Characteristic Automaton

For example:

$$\begin{array}{lcl} E & \to & E + T \quad | \quad T \\ T & \to & T * F \quad | \quad F \\ F & \to & (\, E \,) \quad | \quad \text{int} \end{array}$$

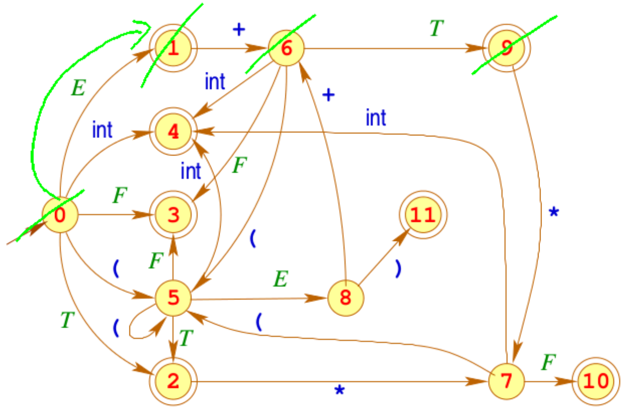


Canonical LR(0)-Automaton

The canonical $LR(0)$-automaton $LR(G)$ is created from $c(G)$ by:

1. performing arbitrarily many ϵ-transitions after every consuming transition
2. performing the powerset construction

... for example:



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Attention:

This parser is only deterministic, if each final state of the canonical $LR(0)$-automaton is conflict free.

LR(0)-Parser

... for example:

$$\begin{array}{lcllcl} q_1 & = & \{[S' \to E\,\bullet], & & & \\ & & \;[E \to E\,\bullet + T]\} & & & \\ q_2 & = & \{[E \to T\,\bullet], & q_9 & = & \{[E \to E + T\,\bullet], \\ & & \;[T \to T\,\bullet * F]\} & & & \;[T \to T\,\bullet * F]\} \\ q_3 & = & \{[T \to F\,\bullet]\} & q_{10} & = & \{[T \to T * F\,\bullet]\} \\ q_4 & = & \{[F \to \text{int}\,\bullet]\} & q_{11} & = & \{[F \to (\,E\,)\,\bullet]\} \end{array}$$

The final states q_1, q_2, q_9 contain more then one admissible item
$\Rightarrow$ non deterministic!

LR(0)-Parser

The construction of the $LR(0)$-parser:

States: $Q \cup \{f\}$ (f fresh)
Start state: q_0
Final state: f

Transitions:

Shift: $(p, a, p\,q)$ if $q = \delta(p, a) \neq \emptyset$
Reduce: $(p\,q_1 \ldots q_m, \epsilon, p\,q)$ if $[A \to X_1 \ldots X_m \bullet] \in q_m$, $q = \delta(p, A)$
Finish: $\boxed{(q_0\,p, \epsilon, \boxed{f})}$ if $\boxed{[S' \to S \bullet] \in p}$

with $LR(G) = (Q, T, \delta, q_0, F)$.

LR(0)-Parser

Correctness:

we show:

The accepting computations of an $LR(0)$-parser are one-to-one related to those of a shift-reduce parser M_G^R.

we conclude:

- The accepted language is exactly $\mathcal{L}(G)$
- The sequence of reductions of an accepting computation for a word $w \in T$ yields a reverse rightmost derivation of G for w

LR(0)-Parser

Attention:
Unfortunately, the $LR(0)$-parser is in general non-deterministic.

We identify two reasons:

Reduce-Reduce-Conflict:
$[A \to \gamma \bullet]\,,\ [A' \to \gamma' \bullet] \in q$ with $A \neq A' \vee \gamma \neq \gamma'$

Shift-Reduce-Conflict:
$[A \to \gamma \bullet]\,,\ [A' \to \alpha \bullet a\,\beta] \in q$ with $a \in T$

for a state $q \in Q$.

Those states are called $LR(0)$-unsuited.